A Theory of Liquidity Spillovers Between Bond and CDS Markets

During the recent debt crisis in Europe, policy makers responded to the controversy surrounding CDS by implementing a series of policies that banned CDS trading. Sambalaibat (2014) documents that a temporary CDS ban increased bond market liquidity but a permanent ban instead decreased bond market liquidity. To explain these patterns, I build a search-theoretic model of OTC bond and CDS markets that features an endogenous liquidity interaction between the two markets and endogenous funding liquidity. I show that these opposing patterns are due to the fact that bond and CDS markets are substitute markets in the short run but are complementary markets in the long run. My results challenge existing theories of liquidity interaction among multiple markets and the common perception that the CDS market is a more liquid market than the bond market.

A large body of work explores, in the context of exchange traded assets, why financial derivatives exist and how they affect the underlying assets. In the context of over-the-counter traded assets, however, we have a limited understanding of the effect of derivatives even though the universe of over-the-counter traded assets is much larger than exchange traded assets. In this paper, I study how derivatives affect liquidity and prices of the underlying assets when they are both traded over-the-counter. I explore this, in particular, in the context of sovereign bond and credit default swap (CDS) markets.

The controversy surrounding CDS during the debt crisis in Europe culminated in a series of policies that banned “naked” purchases of CDS, which is the practice of buying CDS protection without actually owning the underlying government bonds. Sambalaibat (2014) argues that these policies serve as exogenous shocks to the CDS market and allow us to empirically identify the effect of naked CDS trading on the underlying bonds. They document that permanent versus temporary CDS bans had completely opposite effects on bond market liquidity. When the EU voted in October 2011 to permanently ban naked CDS referencing EU countries, countries affected by the

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A buyer of a CDS protection pays a periodic fee until either the contract matures or a default (or a similar event) occurs. In return, the protection seller transfers the purchased amount of insurance in the event of default. The contract specifies the reference entity, the contract maturity date, the insurance amount, and the events that constitute a credit event.
ban experienced a decrease in their bond market liquidity. When Germany temporarily banned naked CDS in May 2010, this pattern reversed: bond market liquidity temporarily increased instead.

To explain these opposing patterns and, consequently, shed light on how CDS markets affect the underlying bond market, I build a dynamic search-theoretic model of over-the-counter (OTC) bond and CDS markets. My model shows that, for investors who want a long exposure to credit risk, bond and CDS markets are substitute markets in the short run but are complementary markets in the long run. Depending on the nature of the ban, as a result, one effect dominates the other. When the CDS ban is temporary, long traders temporarily substitute out of the CDS into the bond market and bond market liquidity temporarily increases. When the ban is permanent, however, as traders are forced to exit the CDS market, they pull out from the bond market also and bond market liquidity decreases.

In the model, I capture the over-the-counter structure of bond and CDS markets using the search and bilateral bargaining mechanism of Duffie, Garleanu, and Pedersen (2005, 2007). A fraction of bond owners are hit by a liquidity shock that requires them to sell their bonds. Locating a buyer, however, involves search costs. When a seller finds a buyer, she takes into account the difficulty of locating a buyer again and ends up selling her bond at a discounted price. Thus, as in the standard search framework, search costs create an illiquidity discount in bond prices.

I study how CDSs affect this illiquidity discount by modeling two novel features. The first is the presence of CDS markets. CDSs are zero net supply derivative assets, while bonds are fixed supply assets, and trading CDS contracts also involves search costs. CDSs exist in the model because they complete markets. In particular, CDSs and bonds pay off in different states (default and non-default states). Buying naked CDS allows short positions with respect to the underlying bonds that are otherwise not possible because investors cannot directly short bonds. This assumption captures a fundamental difference between bond and CDS markets: it is cheaper to short credit risk using the CDS market than using the bond market.

The second novel feature is endogenous funding liquidity. In particular, long investors’ entry rate into the economy endogenously adjusts to the introduction or the shutting down of the CDS market. The result is an endogenous aggregate number of investors who can fund liquidity into both markets. The model thereby features both funding liquidity and market liquidity, and both are endogenous.

In this environment, the complementarity effect works as follows. For investors looking for a long exposure, selling CDSs and buying bonds are two different ways to be exposed to credit risk and they can search for a counterparty simultaneously in both the CDS and the bond market. This ability to simultaneously search in both markets reduces the expected search time of acquiring a long position: long investors now have twice as many potential counterparties and, hence, a higher probability of finding a counterparty in at least one of the two markets. A marginal long investor – who would have been deterred by the search cost when there was just the bond market – now has an incentive to enter both the CDS and the bond market. By
increasing trading opportunities, naked CDS buyers, as a result, relaxes the entry constraints of long investors. And since after entering long investors provide liquidity in both markets, the result is an increased liquidity in the bond market. Thus, trading activity and liquidity in the CDS market spills over to the bond market because there is an increasing returns to scale to searching simultaneously in multiple substitute markets.

Permanently banning naked CDS trading reverses this complementarity effect and eliminates CDS's positive externality on bond market liquidity: long investors are forced to exit the CDS market but, by exiting the CDS market, they pull out from the bond market also. Bond market liquidity, as a result, decreases which is consistent with the observed deterioration in bond market liquidity after the permanent ban.

When the CDS ban is temporary, the benefit of adjusting entry and exit into bond and CDS markets (at the extensive margin) does not outweigh the cost of doing so. As a result, the aggregate number of traders across bond and CDS markets remains unchanged. Instead, there is only a migration at the intensive margin between bond and CDS markets. Long investors – who would have otherwise sold CDSs to the naked CDS buyers – temporarily resort to trading in the bond market by buying bonds and thereby increase liquidity in the bond market. This model implication is consistent with the observed increase in bond market liquidity after the temporary ban.

The model mechanism critically relies on search frictions in the CDS market. Trading frictions in the CDS market create a spillover interaction between bond and CDS markets and thereby help rationalize the empirical patterns. Without search frictions, the existence of naked CDS buyers does not affect bond market liquidity and the CDS market is completely redundant. Also, in the data, transaction costs in sovereign CDS markets are non-trivial: CDS bid-ask spreads are, on average, ten times larger than bond bid-ask spreads. The importance of trading frictions in the CDS market both in the model and in the data challenges the common assumption that the CDS market is a more liquid market. My results show that this is not the case.

The fact that bond and CDS markets can be complementary markets is a novel result in light of existing theoretical studies of liquidity interaction between multiple asset markets. These studies highlight the migration (i.e. the substitution) effect. In these models, the aggregate number of traders across markets is kept fixed and, consequently, introducing additional markets – by construction – results in a fragmentation and a migration of traders across multiple markets. In contrast, I show that endogenizing the aggregate number of investors creates a complementarity between multiple markets and that this interaction helps rationalize the observed empirical patterns.

This paper contributes to the existing literature by providing the first theoretical framework of over-the-counter trading in both the underlying and derivative markets. The recent empirical literature on CDS tries to understand the determinants of the CDS-bond basis. This exercise is confounded by endogeneity problems because liquidity and prices are interdependent within a market as well as across related markets. As my theoretical framework features interdependent bond and CDS market liquidity and
prices, it allows us to disentangle these effects. Thus, I additionally derive testable implications on how exogenous variations in liquidity of bond and CDS markets affect the CDS bond-basis.

This paper highlights a novel mechanism in which naked CDS buyers directly affect liquidity of the underlying bond market. The most commonly posited effect of CDS on the bond market is the “covered” CDS story: the ability to hedge one’s bond portfolio by buying CDS most likely attracts traders into the bond market and increases bond market liquidity. As for the effect of naked CDS trading, a common hypothesis is that it increases liquidity of the CDS market itself and, consequently, indirectly increases bond market liquidity by making CDS a cheaper hedging tool. These effects, however, cannot explain why permanent versus temporary CDS bans affected bond market liquidity differently. This paper instead proposes a theory that rationalizes the opposite effects within the same theoretical environment.

My mechanism is distinct from the effects of basis trades. In a basis trade, investors trade on an arbitrage opportunity that arises if bond and CDS markets price the underlying credit risk differently. For example, if the CDS premium is too low relative to bond yields, the CDS market is underestimating default risk relative to what the bond yields suggest. A basis trading strategy would be to buy the underpriced bonds and hedge their default risk with the currently cheap CDS. Thus, the existence of the CDS market, by creating a potential arbitrage opportunity, may increase the amount of trade and liquidity in the bond market. But basis trades necessarily involve a long position in one market (e.g. buying bonds as in the example) and a short position with respect to the underlying in the other market (e.g. buying CDS). In contrast, in my mechanism, there is an increase in the volume of trade and liquidity in the bond market due to traders seeking a long position with respect to the underlying in both markets.

Finally, the policy implications of my results are that, first, permanently banning naked CDS trading adversely affected bond market liquidity, depressed bond prices, and thereby increased sovereign’s borrowing cost exactly when governments were trying to avert a liquidity dry-up and credit risk spiral. This result is particularly important in the context of a sovereign debt crisis. Second, my results show that temporary versus permanent and anticipated versus sudden regulations can have very different effects.

**Related Literature**

This paper belongs to the search literature of financial assets beginning with the seminal papers Duffie, Garleanu, and Pedersen (2005, 2007). My framework is closely related to the extensions of their environment to multiple assets by Vayanos and Wang (2007), Weill (2008) and, in particular, it is a variant of Vayanos and Weil (2008)’s framework that sheds light on the on-the-run phenomenon of Treasury bonds. I contribute to this literature, first, by modeling over-the-counter trading in derivatives in addition to trading in the underlying asset and, second, by endogenizing the entry decisions of agents into the market for the underlying asset in response to the introduction of the derivative market.

A related paper is Afonso (2011) who endogenizes the entry decisions of
traders but in a single market setting. My model differs by featuring both multiple markets and endogenous entry and therefore sheds light on the rate of entry into one market as a result of introducing another market and on the mechanism through which traders migrate between different markets.

A search theoretic paper applied specifically to CDS markets is Atkeson, Eifeldt, and Weill (2012) who in a static setting study how banks’ CDS exposure arises endogenously depending on their size and their exposure to aggregate risk. In contrast, my paper focuses on naked CDS and studies in a dynamic setting the feedback from the CDS market into the bond market by allowing trade in both the bond and the CDS market as opposed to just the CDS market. Oehmke and Zawadowski (2013) explore how CDS affects bond prices in Amihud and Mendelson (1986) type framework with exogenous trading frictions. In contrast, my model features endogenous trading costs.

A related literature is equilibrium asset pricing models with exogenous trading frictions (see, for example, Amihud and Mendelson (1986), Acharya and Pedersen (2005)). My model features endogenous bond market liquidity and thereby allows for an endogenous interaction and a spillover between the underlying and the derivative markets.


Motivated by the theoretical arbitrage relation between how credit risk is priced through bond prices versus through CDS spreads, a growing number of papers study the joint dynamics of bond and CDS spreads, or equivalently the CDS-bond basis, as well as the relative price discovery mechanism in bond and CDS markets. These papers’ findings suggest that on average the arbitrage relation holds. But when it does not and the price of credit risk in these two markets deviate, where the price discovery takes place (determined by which of the two prices leads the other) is state dependent. In particular, one of the important determinants is the relative liquidity in these markets. I add to this literature by providing a tractable theoretical framework with endogenous liquidity interaction between the two markets and, hence, precise implications on liquidity and prices in both markets.

My work is also related to the literature that studies how CDS affects the issuer of the debt security on which the CDS contracts are written. Empirical
studies include Ashcraft and Santos (2009) and Subrahmanyam, Tang, and Wang (2011), who study the effect on firms’ cost of borrowing and credit risk, respectively. Also Das, Kalimipalli, and Nayak (2013) document that corporate bond market liquidity did not improve with the inception of the CDS market, while Massa and Zhang (2012) and Shim and Zhu (2010) document that CDS markets increased corporate bond market liquidity. In contrast, my paper identifies the effect of naked CDS trading (as opposed to the CDS market in general) on bond market liquidity and focuses on sovereign bond and CDS markets.

On the theoretical front, Arping (2013) and Bolton and Oehmke (2011) formalize the tradeoffs associated with the empty creditor problem in the context of corporate debt and Sambalaibat (2012) in the context of sovereign debt. Duffee and Zhou (2001) find that credit derivatives alleviate the lemons problem associated with banks having private information on their loans. Thompson (2007) and Parlour and Winton (2009) study the tradeoffs that banks face in selling off versus insuring loans on their balance sheets. Thus, these papers have focused on issues surrounding covered CDS buyers who are directly exposed to the issuer’s default risk. This paper instead focuses on how naked CDS buyers affect the issuer’s cost of borrowing through their effect on bond market liquidity and bond prices.

This paper also contributes to the theoretical literature that studies the distribution of liquidity and trade across multiple markets. Examples that use information-based frameworks are Admati and Pfleiderer (1988), Pagano (1989), and Chowdhry and Nanda (1991), while search-theoretic ones are Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008). A typical result in these papers is that traders endogenously concentrate in one market and trade in the other market disappears. Multiple markets can co-exist under additional assumptions of heterogeneous agents and heterogeneous markets so that there is a “cliente” effect. The focus of these papers has been the endogenous cross-sectional distribution of liquidity and trade across markets and assets. This endogeneity is, consequently, on the intensive margin (i.e. the number of traders can vary in the cross-section but the aggregate number of traders is fixed), and the results of these papers are effectively partial equilibrium effects. In my model, if the aggregate number of

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3Ashcraft and Santos (2009) find that CDS has beneficial effects on firms’ cost of borrowing for safer firms but adverse effects for riskier firms as banks may lose the incentive to monitor firms. Subrahmanyam, Tang, and Wang (2011) find CDS increases firms’ credit risk which they attribute to protected creditors’ reluctance to restructure. Berndt and Gupta (2009) find that borrowers, whose loans have been sold off, underperform. Duffee and Zhou (2001) also show that that credit derivatives adversely affect the parallel loan sales market.

5Although I do not formally model the issuer’s borrowing cost in the primary debt markets, He and Milbradt (2012) provide a formal treatment of the feedback loop between credit risk, the issuer’s borrowing cost through the primary debt markets, and liquidity of the secondary bond markets.

6For example, Pagano (1989) shows that if markets differ in their fixed entry cost, then an equilibrium with multiple markets exists and has the following feature: the more liquid market has a larger fixed cost of entry and is also the market where only large traders (those needing a larger portfolio adjustment) are attracted to. This is because the larger market has a bigger absorbing capacity (i.e. minimal price impact) and the fixed entry cost can be spread over a large transaction size.
traders is kept fixed, then (similar to these papers) with the introduction of the CDS market, traders migrate from the bond market to the CDS market and bond market liquidity decreases. However, my model also shows that if the aggregate number of traders is endogenous to the introduction of an additional security (i.e., the endogeneity is on the extensive margin), then the result is the opposite: the number of traders and liquidity in the market for the underlying asset increase.

More broadly, this paper belongs to the literature on the effect of derivatives such as options and futures on the market for the underlying assets. A majority of this literature is empirical. Theoretical frameworks that study the effect of derivatives on liquidity of the underlying asset market include Subrahmanyam (1991), Gorton and Pennacchi (1993), and John, Koticha, Subrahmanyam, and Narayanan (2003) and they also get the “migration” result as the above multiple market information-based models. I add to the literature by endogenizing entry. Also, these papers are based on Kyle (1985) and Glosten and Milgrom (1985) type frameworks where illiquidity arises from asymmetric information. The stylized OTC search framework of my paper is better suited for sovereign bond markets for two reasons. First, trade in sovereign bond markets is fragmented across heterogenous bonds and, second, asymmetric information and insider trading are less severe with respect to governments than with respect to individual firms.

The paper is organized as follows. Section 1 presents the model environment, while Section 2 derives the main theoretical results. Section 3 draws additional testable predictions of the model, Section 4 discusses possible extensions, and Section 5 gives institution details on bond and CDS markets and the bans. Section 6 concludes. All proofs are in the Appendix.

1 Model

Time is continuous and goes from zero to infinity. Agents are risk neutral, infinitely lived, and discount the future at the constant rate \( r > 0 \). There is a bond with supply \( S \) that pays a coupon flow \( \delta_b \). In addition, agents can trade a CDS contract in which a buyer of a CDS contract pays a premium flow \( p_c \) and, in return, benefits from an expected insurance payment of \( \delta_c \). CDS allows both long and short positions to the underlying credit risk: a buyer of a CDS contract has a short exposure, while a seller has a long
exposure. I assume that bonds allow only a long exposure and that agents cannot directly short bonds. The bond coupon flow can be interpreted as an expected coupon flow: with intensity $\eta$ the bond defaults but otherwise pays a dollar of coupon. Hence, $\delta_b = (1 - \eta)\$1$. Similarly, $\delta_c$ can be interpreted as an expected insurance payment: a CDS contract pays out a dollar if there is a default on the coupon payment, thus $\delta_c = \eta \$1$.

Agents’ utility valuations of assets switch randomly between high, average, and low types where each type values the bond and the CDS payoffs as given in Table 1. Let $\theta = 1$ denote a long position (exposed to risk) through the bond or the CDS market, $\theta = 0$ no position, and $\theta = -1$ a short position (i.e. bought CDS). An agent with $\theta_b \in \{0, 1\}$ shares of the bond has a utility flow $\theta_b (\delta_b + x^b_t) - |\theta_b| y$. An agent with a CDS position $\theta_c \in \{-1, 0, 1\}$ has a utility flow $-\theta_c (\delta_c + x^c_t) - |\theta_c| y$, where $x^b_t \in \{-x_b, 0, x_b\}$ and $x^c_t \in \{-x_{ch}, 0, x_{cl}\}$ are stochastic processes, and $y$ is a cost of risk bearing that is positive for both long and short positions. I define an agent with $\{x^b_t = x_b, x^c_t = -x_{ch}\}$ as a high type, with $\{x^b_t = 0, x^c_t = 0\}$ as an average, and with $\{x^b_t = -x_b, x^c_t = x_{cl}\}$ as a low type.

The parameters $x_b$, $x_{ch}$, and $x_{cl}$ capture in a reduced form different reasons traders may have to want to trade bond and CDS with each other (for a similar setup, see Duffie, Garleanu, and Pedersen (2005) for two types of agents and Vayanos and Weill (2008) for three types of agents). One way to interpret them is as hedging benefits. High types may have an idiosyncratic endowment that is negatively correlated with the bond cash flow, while low types have an idiosyncratic endowment that is positively correlated with the bond. Thus, a low type agent would get an extra disutility of $x_b$ from holding the bond ($\theta_b = 1$), while a high type would get an extra utility $x_b$. If a low type sells CDS ($\theta_c = 1$), she experiences a greater disutility paying out the insurance payment $-(\delta_c + x_{cl}) - y$ than if she were a high type $-(\delta_c - x_{ch}) - y$. Conversely, if a low type buys CDS ($\theta_c = -1$), he benefits more from the insurance payment $((\delta_c + x_{cl}) - y)$ than if he were a high type $((\delta_c - x_{ch}) - y)$.

Appendix A.1 gives an example of how the parameters $x_b$, $x_{ch}$, and $x_{cl}$ can be functions of the default intensity of the bond. In the Online Appendix, I show in an environment with risk averse agents the hedging benefits are functions of the risk aversion parameter, the correlation between agents’ idiosyncratic endowment and the bond cash flow, and the riskiness of the bond.

The online appendix is available at https://sites.google.com/site/sambalaibat/.
Table 1: Valuation of bond and CDS payments by high, average, and low type agents.

Agents are heterogeneous in their valuation of bond and CDS cash flows. As shown in the “Bond Owner” column, high type agents derive a higher utility from a long exposure to the bond, while low type agents derive a disutility from a long exposure to the bond. Conversely, low type agents derive a higher utility from a short position (as shown in the “CDS Buyer” column), while high type agents derive a disutility from a short position. As a result, in equilibrium high type agents search for long positions, while low type agents short credit risk. Average type agents are in between.

<table>
<thead>
<tr>
<th>Types</th>
<th>Bond Owner ($\theta_b = 1$)</th>
<th>CDS Buyer ($\theta_c = -1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$\delta_b + \delta_b - y$</td>
<td>$\delta_c - x_{ch} - y$</td>
</tr>
<tr>
<td>Average</td>
<td>$\delta_b - y$</td>
<td>$\delta_c - y$</td>
</tr>
<tr>
<td>Low</td>
<td>$\delta_b - x_b - y$</td>
<td>$\delta_c + x_{cl} - y$</td>
</tr>
</tbody>
</table>

**Assumption 1.** $x_{ch} + x_{cl} > 2y > x_{ch}$

Assumption 1 ensures that low valuation agents will want to short by buying CDS, while average types will not want to short. To see this, if a low type agent buys CDS from a high type, the buyer’s flow surplus from the transaction is $(\delta_c + x_{cl}) - y - p_c$, while the seller’s is $p_c - (\delta_c - x_{ch}) - y$. The total surplus is then $x_{ch} + x_{cl} - 2y$, which is positive from Assumption 1. If, instead, an average type buys CDS from a high type, the total surplus is $x_{ch} - 2y$, which is negative from Assumption 1. Thus, the cost of risk bearing parameter, $y$, helps to restrict the set of possible trades between different agents and arises in a setting with more than two types of agents. The difference in valuations between agents have to be far enough to offset this cost of risk bearing and, consequently, it is the high type agents (and not the average type agents) that will want to sell CDS to low types. In a model with risk averse agents, as shown in Vayanos and Weill (2008), this parameter will be a function of the risk aversion parameter and the riskiness of the asset.

There is an infinite mass of average valuation agents. A fixed flow $2F_h$ of average types switch to a high type, and a flow $F_l$ switch to a low type. A high type agent enters to trade in the bond and the CDS market only if the expected value of trading as a high type (denoted by $V_{hn}$) is at least greater than the value of her outside option. Among the agents that switch to a high type, I assume half of them do not have an outside option and hence always enter. The other half has a positive opportunity cost of entering denoted by $O_h$ and fraction $\rho$ of them enter according to

$$\rho = \begin{cases} 
1 & V_h(n) > O_h \\
[0, 1] & V_h(n) = O_h \\
0 & V_h(n) < O_h.
\end{cases}$$

(1)

Thus, the total flow of high types actually entering is $(1 + \rho)F_h$. Conversely, high types switch to an average type with Poisson intensity $\gamma_d$, while low

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Afonso (2011) provides a more general setup in which there is a continuous distribution of agents with different outside values. My setup is a special case of this.

The assumption that a portion of high types are always entering is for simplicity and is a way to scale up the measure of high types in the economy so that even if $\rho = 0$, the steady state measure of high types is greater than the steady state measure of low types and the bond supply. This simplifies the derivation of existence and uniqueness of the steady state equilibrium without affecting the main channels of the model.
types switch to an average type with Poisson intensity $\gamma_u$. Thus, the steady state measure of high types is at least $\frac{F_h}{\gamma_d}$, while the steady state measure of low type agents is $\frac{F_l}{\gamma_w}$.

**Assumption 2.** $\frac{F_h}{\gamma_d} > S + \frac{F_l}{\gamma_w}$

Assumption 2 ensures that high types are the marginal investors in the bond.

### 1.1 The Bond and the CDS Market

Buyers and sellers in the bond market meet at a rate $\lambda_b \tau_{bb} \tau_{bs}$, where $\lambda_b$ is the exogenous matching efficiency of the bond market, and $\tau_{bb}$ and $\tau_{bs}$ are the measures of bond buyers and sellers, respectively. Given the total meeting rate, buyers find a seller with intensity $q_{bs} \equiv \lambda_b \tau_{bs}$, and sellers find a buyer with intensity $q_{bb} \equiv \lambda_b \tau_{bb}$. Once matched, a buyer and a seller Nash-bargain over the price so that the buyer gets a fraction $\phi$ of the total gains from trade and the seller gets the remaining surplus.

Analogously, in the CDS market, CDS buyers find a seller with intensity $q_{cs} \equiv \lambda_c \tau_{cs}$, and sellers find a buyer with intensity $q_{cb} \equiv \lambda_c \tau_{cb}$, where $\tau_{cb}$ and $\tau_{cs}$ are the measures of CDS buyers and sellers, respectively.

### 1.2 Agent Types and Transitions

Table 2 shows the various types and their possible asset positions. The variable $\mu_\tau$ denotes the measure of type $\tau \in T$ agents, where $T \equiv \{hn, ln, hob, aob, hoc, aoc, lsc\}$ is the set of agent types. Agent types $hn$ and $ln$ are high and low non-owners, $hob$ and $aob$ are high and average bond owners, $hoc$ and $aoc$ are high and average types that have sold CDS, and $lsc$ are low types who have bought CDS.

<table>
<thead>
<tr>
<th>$(\theta_b, \theta_c)$</th>
<th>$(0,0)$</th>
<th>$(1,0)$</th>
<th>$(0,1)$</th>
<th>$(0,-1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$\mu_{hn}$</td>
<td>$\mu_{hob}$</td>
<td>$\mu_{hoc}$</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>$\infty$</td>
<td>$\mu_{aob}$</td>
<td>$\mu_{aoc}$</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>$\mu_{ln}$</td>
<td></td>
<td>$\mu_{lsc}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1 shows the transitions between types. High types (the top row) want a long exposure to the underlying credit risk by either buying a bond or selling CDS. If they switch to an average type, they will try to liquidate their existing long position by selling the bond or exit the economy if they did not have any existing positions. Average types (the middle row) do not
want a long or a short exposure so they just stay out of the markets. Low types (the bottom row) want a short exposure by buying CDS.

Since a high type non-owner \((hn)\) wants a long exposure to credit risk, he will search to buy a bond or to sell CDS and will find counterparties with intensities \(q_{bs}\) and \(q_{cb}\), respectively. Before he is even able to find a counterparty, he may switch to an average type and exit the economy. But if he finds and trades with a bond-seller, he becomes a high type bond owner, \(hob\). He is happy to keep that position until he is hit by a liquidity shock, in which case he becomes an average type and will want to liquidate his bond position (that is, become a bond seller \((aob)\)). Upon finding a bond buyer, he exits the market.

A high non-owner \((hn)\) could also sell CDS (which occurs with intensity \(q_{cb}\)) and become a \(hoc\) type who has a long-exposure to credit risk. He is happy with this long exposure unless he switches to an average type and becomes one of \(aoc\). As an average type, instead of remaining a CDS seller, he will try to unwind his position by searching for another CDS seller to take over his side of the trade at the original price. In practice, this is called assignment or novation.

Since a low non-owner \((ln)\) wants to short credit risk, she searches to buy CDS, finds a counterparty with intensity \(q_{cs}\) and consequently becomes a CDS holder, \(lsc\). If she switches to an average type, she terminates her contract, while her counterparty reverts back to an \(hn\) type and has to start over his search.
Figure 1: Transitions Between Agent Types

The figure shows the transitions between agent types. A flow of \((1 + \rho)F_h\) agents enter the economy as high types and flow \(F_l\) as low types. High type agents are hit with a liquidity shock (and become an average valuation) with intensity \(\gamma_d\). Conversely, low types switch to an average type with intensity \(\gamma_u\). A trader seeking a long position \((hn)\) finds a counterparty in the bond and the CDS market with probabilities \(q_{bs}\) and \(q_{cb}\), respectively. A bond seller, \(aob\), finds a buyer with probability \(q_{bb}\). A trader seeking to establish a short position, \(ln\), by buying CDS finds a counterparty with probability \(q_{cs}\).

\[
\begin{align*}
(1 + \rho)F_h & \quad \gamma_d \\
\mu_{hn} & \quad q_{bs} \quad \mu_{hob} \quad \mu_{hoc} \\
\infty & \quad q_{bb} \quad \mu_{aob} \quad \mu_{aoc} \\
F_l & \quad \gamma_u + q_{cs} \\
\mu_{ln} & \quad q_{cs} \quad \mu_{lsc} \\
(0, 0) & \quad (1, 0) \quad (0, 1) \quad (0, -1)
\end{align*}
\]

Given the search choices of agents, the measure of buyers and sellers in the bond and CDS markets are: \(\tau_{bb} = \mu_{hn}\), \(\tau_{bs} = \mu_{aob}\), \(\tau_{cs} = \mu_{hn}\), \(\tau_{cb} = \mu_{hn} + \mu_{aoc}\). In the steady state, the measures of types are constant and the in-flow of agents has to equate the out-flow for each type as shown in Table 3.

**Table 3: Flow-ins and outs**

In the steady state equilibrium, the measure of agent types is constant: a flow of agents turning into a particular type (Flow-in) has to equal the flow of agents switching out of that type (Flow-out).

| Type    | Flow-in = Flow-out: | \[
\begin{align*}
\mu_{hn} & \quad (1 + \rho)F_h + \gamma_u \mu_{hoc} = \gamma_d \mu_{hn} + (q_{bs} + q_{cb}) \mu_{hn} \\
\mu_{ln} & \quad F_l = \gamma_u \mu_{hn} + q_{cs} \mu_{hn} \\
\mu_{hob} & \quad q_{bs} \mu_{hn} = \gamma_d \mu_{hob} \\
\mu_{aob} & \quad \gamma_d \mu_{hob} = q_{bb} \mu_{aob} \\
\mu_{hoc} & \quad q_{cb} \mu_{hn} = \gamma_d \mu_{hoc} + \gamma_u \mu_{hoc} \\
\mu_{aoc} & \quad \gamma_d \mu_{hoc} = \gamma_u \mu_{aoc} + q_{cs} \mu_{aoc} \\
\mu_{lsc} & \quad q_{cs} \mu_{hn} = \gamma_u \mu_{lsc}
\end{align*}
\]

Bond market clearing imposes that the total measure of bond owners has to equal the bond supply:

\[
\mu_{hob} + \mu_{aob} = S. \quad (2)
\]
CDS market clearing requires that the total number of CDS contracts sold has to equal the number of CDS contracts purchased:

\[ \mu_{hoc} + \mu_{aoc} = \mu_{lsc}. \] (3)

1.3 Prices and Bargaining

Prices of bonds and CDS arise from bilateral bargaining between buyers and sellers. Let \( V_\tau \) denote the expected utility of type \( \tau \in \mathcal{T} \). A bond buyer’s marginal benefit of buying a bond is the increase in his expected utility \( V_{hob} - V_{hn} \), and his marginal cost is the bond price \( p_b \). Thus, he is willing to buy as long as the marginal benefit is greater than the marginal cost: \( V_{hob} - V_{hn} \geq p_b \), and the smaller the price is, the larger is his surplus. Analogously, for a seller, the marginal benefit of selling her bond is the bond price, \( p_b \), and in return she is giving up the value of being a bond owner, \( V_{aob} \), which is the marginal cost. Hence, she will sell as long as \( p_b \geq V_{aob} \).

Thus, the bond price has to lie in the interval: \( V_{aob} \leq p_b \leq V_{hob} - V_{hn} \) and the length of this interval is the total surplus from trade. The buyer and the seller split the surplus proportional to their respective bargaining powers: \( \phi \) and \( 1 - \phi \). The greater the bargaining power of the buyer (i.e. higher \( \phi \)), the lower the bond price:

\[ p_b = \phi V_{aob} + (1 - \phi) (V_{hob} - V_{hn}). \] (4)

Analogously, a CDS seller and a CDS buyer Nash-bargain over the price such that the seller and the buyer get \( \phi \) and \( 1 - \phi \) fractions of the total surplus, respectively. The buyer’s surplus is \( V_{lsc} - V_{ln} \) and the seller’s is \( V_{hoc} - V_{hn} \). Thus, the CDS price is implicitly defined by:

\[ V_{hoc} - V_{hn} = \phi (V_{lsc} - V_{ln} + V_{hoc} - V_{hn}). \] (5)

A CDS seller who switches to an average, \( aoc \), will search for another CDS seller to take over his side of the trade (at the original price) and exit with zero utility if \( 0 - V_{aoc} > 0 \).

1.4 Value Functions

To determine the expected utilities of types, consider, for example, an \( hn \) type. In a small time interval \([t + dt]\), he could (a) switch to an average valuation (with probability \( \gamma_{d} dt \) and get utility 0), (b) become a bond owner (with probability \( q_{b} dt \) and get \( V_{hob} - p_b \)), (c) become a CDS seller (with probability \( q_{c} dt \) and get utility \( V_{hoc} \)), or (d) remain an \( hn \) type:

\[ V_{hn} = (1 - r dt) \left( \gamma_{d} dt (0) + q_{b} dt (V_{hob} - p_b) + q_{c} dt V_{hoc} + (1 - \gamma_{d} dt - q_{b} dt - q_{c} dt) V_{hn} \right). \]

After simplifying and taking the continuous time limit, we get:

\[ r V_{hn} = \gamma_{d} (0 - V_{hn}) + q_{b} (V_{hob} - p_b - V_{hn}) + q_{c} (V_{hoc} - V_{hn}). \] (6)

The flow value equations for the other types are analogously derived and are shown in Appendix A.
1.5 Equilibrium

Definition 1. A steady state equilibrium is given by types’ measures \( \{ \mu_\tau \}_{\tau \in T} \), prices \( \{ p_b, p_c \} \), entry decisions \( \{ \rho \} \), and value functions \( \{ V_\tau \}_{\tau \in T} \) such that:

1. \( \{ \mu_\tau \}_{\tau \in T} \) solve the steady state in-flow and out-flow equations in Table 3.

2. Market clearing conditions (2) and (3) hold.

3. Entry decisions, \( \{ \rho \} \), solve (1).

4. Bond and CDS prices, \( \{ p_b, p_c \} \), solve (4) and (5).

5. Agents’ value functions, \( \{ V_\tau \}_{\tau \in T} \), solve agents’ optimization problem given by (6), and (A.14)–(A.19).

In a dynamic search model with multiple assets and more than two types of agents, we can solve for the equilibrium only numerically and the existence of an equilibrium cannot be established for general parameter values (see Vayanos and Weill (2008) and Weill (2008)). However, Vayanos and Weill (2008) and Weill (2008) show that the existence of an equilibrium can be established when search frictions are small. Following their methodology, I show in the next proposition that a unique steady state equilibrium exists when search frictions are small (that is, \( \lambda_b \) and \( \lambda_c \) are large).

Proposition 1. Suppose

\[
x_b = \left( \frac{x_{cl} - (x_{cl} - 2y) \left( \frac{q_a + \gamma_a + \gamma_d}{q_a + \gamma_a + \gamma_d} \right)}{\gamma_a + \gamma_d + \gamma_b + \gamma_c} \right) > 0.
\]

Then, for large \( \lambda_b \) and \( \lambda_c \), there exists a unique equilibrium.

The proof is given in Appendix A. The proof of uniqueness involves the following steps. Given \( \rho \), Appendix A shows that the set of equations that characterizes the dynamics of the population measures together with the market clearing conditions has a unique solution. Given this solution to the population measures, a linear system of equations characterizing the agents’ value functions and prices uniquely determines \( \{ V_\tau \}_{\tau \in T} \). Thus, for any \( \rho \in [0, 1] \), \( V_{hn} \) is uniquely determined. The agent’s entry decision can be either an interior solution or one of two corner solutions (\( \rho = 0 \), \( \rho = 1 \)). To show that the agents’ entry decision has a unique solution, the Appendix shows that if (7) holds, \( V_{hn} \) is a strictly decreasing function of \( \rho \).

To show the existence of an equilibrium, I verify that all the conjectured optimal trading strategies are indeed optimal. In particular, I first show that the total surplus from trading the bond is positive: \( \omega_b = V_{hob} - V_{hn} - V_{aob} > 0 \). By construction, this will ensure that a high type agent will optimally choose to buy a bond, while an average type will not want to be a bondholder and, if she had previously purchased a bond, she will prefer to sell it. Second, Appendix A shows that the total surplus from trading CDS is positive: \( \omega_c = V_{hoc} - V_{hn} + V_{lsc} - V_{ln} > 0 \). This ensures that high type agents will want to sell
CDS, while low type agents will want to buy CDS. Third, I verify that the average type agents will prefer to stay out of the markets completely instead of being a CDS buyer or a CDS seller: \(0 - V_{\text{aoc}} > 0\). The latter ensures that an agent who had previously been a high type agent and had sold CDS will prefer to find another seller to take over her side of the trade (at the original CDS price) and exit with zero utility.

2 Theoretical Results

To fix ideas, I will be interchangeably referring to buyers in the bond market (\(\mu_{hn}\)) as liquidity providers. They also fund liquidity into the CDS market by selling CDS. Conversely, liquidity demanders in the bond market are the bond sellers (\(\mu_{aob}\)) and in the CDS market are the CDS buyers (\(\mu_{ln} + \mu_{aoc}\)). The measures of these liquidity demanders and providers arise endogenously depending on the endogenous entry decision \(\rho\), the efficiency of the matching functions \(\{\lambda_b, \lambda_c\}\), the parameters that determine flows into the economy \(\{F_h, F_l\}\), and the transition intensities between different valuations, \(\{\gamma_d, \gamma_u\}\).

**Proposition 2.** *If the bond market is frictionless (\(\lambda_b \to \infty\)), the bond price is given by*

\[
p_b = \frac{\delta_b + x_b - y}{r} \tag{8}
\]

*and the CDS market does not affect the bond market.*

Proposition 2 shows that, without search frictions in the bond market, the bond price is given by the present value of high types' valuation of the bond. A bond owner – upon getting a liquidity shock – can sell instantly to another high type trader. As a result, bonds are always in the hands of high type agents and never held by agents who have a lower valuation. From (4) the bond price is the weighted average of the marginal valuations of different types of bond owners. Since high types are the only bond holders, the bond price is given by their valuation only. In this frictionless environment, the CDS market does not affect the bond market.

**Proposition 3.** *The bond price is given by:*

\[
p_b = \frac{\delta_b + x_b - y}{r} - \left(\frac{\gamma_d x_b}{rK} + \phi(q_{bs} + r)\frac{x_b}{rK}\right) q_{ch} \Delta_{hoc}, \tag{9}
\]

*where*

\[
\Delta_{hoc} \equiv \frac{\phi(-\phi q_{ba} x_b + k x_c)}{r + \gamma_d + \gamma_u + q_{cs}(1 - \phi) + \phi q_{ch} k - \phi q_{cb} q_{bs} \phi}, \tag{10}
\]

\[
k \equiv r + \gamma_d + q_{ba} \phi + q_{bb}(1 - \phi).
\]

Proposition 3 shows that with search frictions in the bond market the bond price is lower than the frictionless price in (8). The intuition is as follows. A bond owner – who gets a liquidity shock and has to sell her bond – faces a difficulty of locating a counterparty. She is stuck with a bond that
she gets a disutility from. When she does find a buyer, she takes into account the difficulty of locating a buyer again and ends up selling at a discounted price. Similarly, a potential bond buyer is only willing to buy at a low price because he anticipates this trading friction when it is his turn to liquidate his bond position.

Thus, search costs create an illiquidity discount in the bond price given by the difference between (9) and the frictionless price (8): the sum of the second and the third terms in (9). In particular, the third term is the additional discount in the bond price due to bond buyers having an outside option of providing liquidity in the CDS market (that is, by selling CDS).

**Definition 2.** The illiquidity discount, $d$, in the bond price is defined by the difference between the frictionless bond price (8) and the bond price with search frictions present in the bond market (9):

$$
\begin{align*}
    d &\equiv \gamma d_x b + \phi (q_{bs} + r) \frac{x_b}{rK} + \frac{(q_{bb} + r)(1 - \phi)}{rK} q_{cb} \Delta_{hoc}.
\end{align*}
$$

### 2.1 The Effect of CDS on Bond Market Liquidity

The next proposition gives the main theoretical result of the paper. It shows that shorting bonds by buying (naked) CDS increases bond market liquidity and that bond and CDS markets are complementary markets. Figure 2 illustrates the result.

**Proposition 4 (The Complementarity Effect).** In the equilibrium of Proposition 1, there exists $\lambda_c > 0$ such that for all $\lambda_c > \lambda_c$,

$$
\begin{align*}
    d(\lambda_c) &\leq d^{no\mbox{-}cds}.
\end{align*}
$$

The proof is given in Appendix A and the mechanism consists of the following parts. First, for a given rate of entry, $\rho$, the introduction of the CDS market increases the value of entering the economy as a high type agent. As shown in Figure 3, $V_{hn}(\rho)$ shifts up to the solid red line. For traders looking to acquire a long position (i.e. high type agents), selling CDSs and buying bonds are two different ways to be exposed to credit risk and they can search for a counterparty simultaneously in both the CDS and the bond market. This ability to simultaneously search in both markets reduces the expected search time of acquiring a long position in either market. In particular, long traders now have twice as many potential counterparties and, hence, a higher probability of finding a counterparty in at least one of the two markets than if there were just the bond market. A marginal long trader – who otherwise

---

12The threshold $\lambda_c$ is such that, for $\lambda_c < \hat{\lambda}_c$, the equilibrium entry rate $\rho \in [0, 1]$ is given by the corner solution of $\rho = 1$. The intuition is that if search frictions in the CDS market worsen (i.e. $\lambda_c$ becomes even smaller), a greater fraction of high type agents would want to enter and provide liquidity in the CDS market. But at most $\rho = 1$ fraction of high type agents is able to enter, and being at the corner solution is analogous to the entry rate being fixed. A numerical calibration (available upon request) shows that for reasonable parameter ranges of $\lambda_c$ the equilibrium entry rate will be given by an interior solution. See [Vayanos and Weill (2008)] for a calibration exercise and for a discussion on reasonable magnitudes for $\lambda_c$'s.
would have been deterred by the search cost in the bond market – now has a greater incentive to enter and search in both the CDS and the bond market.

Long investors enter until the marginal entrant is again indifferent between entering or not (that is, until \( V_{hn} \) crosses the outside option \( O_h \)). The result is an increase in the equilibrium number of high-valuation investors and consequently in the aggregate funding liquidity. As shown in Figure 3, the equilibrium entry rate increases from \( \rho^{ncds} \) to \( \rho^{cds} < \infty \).

Third, in the presence of search frictions in the CDS market, the increase in the aggregate number of high-valuation investors is strictly greater than the total demand for CDS (\( F \)). New entrants are held up searching for a long position instead of selling CDS immediately upon entry. This translates to an increased flow of traders still searching for a long position and hence an increase in the number of bond buyers. Bond sellers, in turn, are able to find a buyer more quickly, and bond market liquidity and the bond price increase.

Figure 5 illustrates that, in the presence of the CDS market, there is a greater number of bond buyers, fewer bond sellers, and a larger volume of trade in the bond market.

2.1.1 Model Implication on a Permanent CDS Ban

The above results showed that bond and CDS markets are complementary markets: the existence of naked CDS buyers increases bond market liquidity. This is a long-term effect. Permanently banning naked CDS buyers will reverse this positive effect and adversely affect bond market liquidity. Specifically, if the number of naked CDS buyers permanently decreases for an exogenous reason (as happened with the permanent EU ban), long investors – who would have been the counterparties to the naked CDS buyers – are forced to exit the CDS market. But by exiting the CDS market, they exit the bond market also. As a result, bond market liquidity and bond prices decrease. This model prediction is consistent with the observed decrease in bond market liquidity after the permanent ban.

2.1.2 The Importance of CDS Search Frictions

CDS has opposing complementary versus substitution effects on bond market liquidity. The substitution effect arises from bond buyers having an outside option of providing liquidity in the CDS market (by selling CDS). This effect depresses the bond price and lowers bond market liquidity. Thus, the increase in the number of high-valuation investors due to CDS has to be large enough for the complementary effect to more than offset the substitution effect.

By how much the CDS market increases the entry rate depends on the matching efficiency of the CDS market as illustrated in Figure 4. One extreme is a frictionless CDS market (\( \lambda_c \to \infty \)). In this case, the complementary and

\[ \text{Each additional long entrant increases competition and lowers the expected utility of the other high type agents. As shown in Figure 3, } V_{hn}(\rho) \text{ is decreasing in } \rho. \]

\[ \text{Recall in 9 the additional discount in the bond price due to CDS.} \]
the substitution effects exactly offset one another. The increase in the equilibrium measure of all high-valuation investors \((\rho^{cds} - \rho^{nocds}) F_{h}^{cds}\), is exactly equal to the total demand for CDS (the measure of all low types, including those who have purchased CDS: \(F_{lu} = \mu_{ln} + \mu_{lsc}\)). This is because liquidity providers sell CDS immediately upon entry to the flow of CDS buyers. As a result, introducing the CDS market increases the aggregate number of high-valuation investors, but this increase does not translate to an increase in the number of bond buyers as illustrated in Figure 5. Thus, as shown in Proposition 5 if the CDS market is frictionless, the CDS market is redundant and has no effect on the bond market.

**Proposition 5.** \(\lim_{\lambda_{c} \to \infty} d(\lambda_{c}) = d^{nocds}\).

So only in the presence of search frictions in the CDS market, allowing short positions through the CDS market has a positive externality on the bond market. Thus, frictions inherent in OTC markets create an interaction between bond and CDS markets and thereby help explain the empirical patterns. With search frictions in the CDS market, for investors who will trade as a counterparty to the short traders, there is effectively an increasing returns to scale by searching simultaneously in multiple markets. One way to interpret this result is that the costly research effort incurred in pricing individual bonds helps an investor price CDS relatively quickly and vice versa.

### 2.1.3 The Importance of Endogenous Entry

If entry was exogenous, there would only be a substitution effect. In this case, the introduction of the CDS market shrinks the size of the bond market: some investors who would have otherwise bought bonds migrate to the CDS market and sell CDS instead. Existing bond sellers effectively compete with CDS buyers for the same set of traders that can provide liquidity in either market. Due to a fewer number of bond buyers, bond sellers face greater congestion externality and greater search costs. Thus, with exogenous entry, the effect of the CDS market is on the intensive margin only: the aggregate number of market participants is fixed, and there is only a migration or a substitution between the bond and the CDS market.

With endogenous entry, on the extensive margin, a larger number of investors flow into both the bond and the CDS market. The increase in the entry rate more than offsets the substitution effect: it replaces the bond buyers that migrated to the CDS market and, due to search frictions in the CDS market, results in an even greater number of potential bond buyers.

### 2.2 A Temporary Naked CDS Ban

So far, I have compared the steady-state bond prices and bond market liquidity in settings with and without CDS markets while allowing for the aggregate number of traders to change between these steady states. This analysis can speak to the effect of a permanent CDS ban. In this section, I consider instead the immediate impact of a temporary CDS ban.
I model a temporary naked CDS ban as a one-time unexpected drop in the number of naked CDS buyers. To focus on the immediate impact of the shock, I assume that the flow of entrants remains fixed in the short run as the economy rebounds back to the steady state equilibrium. Time can be relabeled so that \( t = 0 \) corresponds to the time at which this shock occurs. As the shock hits, the distribution of the measure of types switches to \( \{\mu_\tau(0)\}_{\tau \in \mathcal{T}} = \{\hat{\mu}_\tau\}_{\tau \in \mathcal{T}} \). I define \( \{\hat{\mu}_\tau\}_{\tau \in \mathcal{T}} \) so that all its elements are equal to the steady state measure of types, except the measure of naked CDS buyers is set zero: \( \hat{\mu}_{hn} = 0 \).

The time-varying equilibrium measure of \( hn \) type agents from this shock back to the steady state is given by the solution to the following ODE:

\[
\dot{\mu}_{hn}(t) = (1 + \rho) F_h + \gamma_a \mu_{hoc}(t) - [\gamma_d \mu_{hn}(t) + (q_{bs}(t) + q_{cb}(t)) \mu_{hn}(t)],
\]

where the initial condition is given by \( \{\mu_\tau(0)\}_{\tau \in \mathcal{T}} = \{\hat{\mu}_\tau\}_{\tau \in \mathcal{T}} \) and the entry rate \( \rho \) is kept fixed at the steady state level. The dynamics for the measures of other agents are analogously characterized in (A.49)–(A.55).

Agent \( hn \)’s value function evolves according to:

\[
\dot{V}_{hn}(t) = rV_{hn}(t) - [\gamma_d (0 - V_{hn}(t)) + q_{bs}(t) (V_{hoc}(t) - V_{hn}(t)) - p_h(t)] + q_{cb}(t) (V_{hoc}(t) - V_{hn}(t))],
\]

where

\[
p_h(t) = \phi V_{aob}(t) + (1 - \phi) (V_{hoc}(t) - V_{hn}(t))
\]

and

\[
V_{hoc}(t) - V_{hn}(t) = \phi (V_{lsc}(t) - V_{hn}(t)) + V_{hoc}(t) - V_{hn}(t).
\]

It is analogous for the other agents as shown in (A.56)–(A.62). Define \( \Delta_{hob} \equiv V_{hob} - V_{hn}, \omega_b \equiv V_{hob} - V_{hn} - V_{aob}, \) and \( \omega_c \equiv V_{hoc} - V_{hn} + V_{lsc} - V_{ln} \). Then, we can rewrite all the ODEs for the value functions in terms of \( \Delta_{hob} \), \( \omega_b \) and \( \omega_c \). For example,

\[
\dot{V}_{hn}(t) = rV_{hn}(t) - [\gamma_d (0 - V_{hn}(t)) + q_{bs}(t) \phi \omega_b(t) + q_{cb}(t) \phi \omega_c(t)].
\]

In turn, solutions for \( \Delta_{hob} \), \( \omega_b \) and \( \omega_c \) are given in Proposition 6.

**Proposition 6.** Given the solution to the time-varying dynamics of agent measures, the dynamics for \( \Delta_{hob} \) and \( V_{aob} \) are given by:

\[
\Delta_{hob} = \frac{\delta_b + x_b - y}{r} - \int_t^\infty e^{-r(s-t)} ((\gamma_d + q_{bs} \phi) \omega_b + q_{cb} \phi \omega_c) \, ds,
\]

\[
V_{aob} = \frac{\delta_b - y}{r} + \int_t^\infty e^{-r(s-t)} q_{bb}(1 - \phi) \omega_b \, ds,
\]

where

\[
\begin{bmatrix}
\omega_b(t) \\
\omega_c(t)
\end{bmatrix} = \int_t^\infty e^{-f_u^t A(u) \, du} \begin{bmatrix}
x_b \\
x_d + x_{ch} - 2y
\end{bmatrix} \, ds,
\]

\[
A(t) = \begin{bmatrix}
\delta_b + x_b & q_{bs}(1 - \phi) \\
q_{bs} \phi & \frac{q_{cb} \phi}{r} + q_{cs}(1 - \phi)
\end{bmatrix}.
\]
2.2.1 Model Implication on a Temporary CDS Ban

Figures 6 and 7 plot the transition dynamics of types’ measures and of the bond illiquidity discount from the CDS ban at $t = 0$ back to the steady state. The sudden drop in the number of naked CDS buyers frees up long investors who would have otherwise sold CDS. These long investors temporarily substitute trading in the CDS market with trading in the bond market (as bond buyers). In turn, bonds sellers temporarily benefit from the ban as they now locate bond buyers more quickly and face lower search costs. In the short term, bond and CDS markets are substitute markets.

Thus, when the CDS market is shut down temporarily, the immediate effect is an increase in bond market liquidity. This prediction is consistent with the observed increase in bond market liquidity following the temporary German ban as documented in Sambalaibat (2014). In the short term, the substitution effect dominates: long investors do not exit at the extensive margin but instead resort to temporarily trading in the bond market.

2.2.2 An Implicit Cost of Entry

The substitution effect arises because long traders end up temporarily trading in the bond market instead of exiting entirely from both markets (that is, exiting at the extensive margin). I arrive at this result by keeping the entry rate fixed, which is a reduced form way to capture an adjustment cost of entry.

Although I do not explicitly incorporate an adjustment cost of entry, equation (11) illustrates one possible way of incorporating it. Now, in addition to comparing the value of entering $V_h(n)$ with the outside investment opportunity $O_h$, high type agents have to take into account a cost of entry, $c(n)$, that varies with the entry rate:

$$
\rho = \begin{cases} 
1 & V_h(n) - c(n) > O_h \\
[0, 1] & V_h(n) - c(n) = O_h \\
0 & V_h(n) - c(n) < O_h 
\end{cases}
$$

where $c'(n) \geq 0$, $c''(n) > 0$, $c(0) \geq 0$, and $c(1) < \infty$.

Figure 8 figuratively illustrates an example of such a cost function. The temporary CDS ban leads to a small decrease in the value of trading as a high type. When the scale of entry is already large and due to the convexity of $c(n)$, a tiny decrease in $\rho$ results in a large decrease in the cost. As a result, the entry rate does not have to change as much in response to a temporary ban. In contrast, with a permanent ban, the value of trading as a high type decreases by a lot. In addition, due to the convexity, as the entry rate $\rho$ decreases, the resulting decrease in the cost of entry becomes less responsive. As a result, the entry rate has to decrease by a lot in response to a permanent ban. We can also back out how the short-run dynamics of the cost of entry has to look like from the dynamics of $V_h(n)$ which is shown in Figure 9.

\[\text{As the ban is lifted, the number of traders searching to buy CDS increases until the fraction of CDS buyers who finds a CDS seller equals the flow of new low type agents entering the economy.}\]
3 Additional Empirical Predictions

In this section, I analyze the comparative statics of the model and highlight additional empirical predictions. I tease out how exogenous variations in bond and CDS market liquidity affect the endogenous liquidity and prices in both markets and consequently the CDS-bond basis.

I start by characterizing closed-form solutions for the CDS price and CDS market liquidity.

**Proposition 7.** The price of a CDS contract is given by:

\[
p_c = \delta_c + x_{cl} - y - \frac{1 - \phi}{\phi} (r + \gamma_u + q_{cs}) \Delta_{hoc}
\]

where \( \Delta_{hoc} \) is given in (10).

**Proposition 8.** The CDS price in a frictionless environment \((\lambda_c, \lambda_b \to \infty)\) is given by:

\[
\lim_{\lambda_c, \lambda_b \to \infty} p_c = \delta_c - x_{ch} + y
\]

*Intuition.* High-valuation investors are the marginal sellers of CDS contracts. In a frictionless environment, the CDS price, as a result, is given by the high-valuation investors’ flow cost of providing insurance.

It is straightforward to show that CDS contracts are more expensive with search frictions than without: \( p_c > \lim_{\lambda_c, \lambda_b \to \infty} p_c \). Analogous to the definition of bond market illiquidity, we define CDS market illiquidity as the wedge between the CDS price with search frictions versus the price without frictions. In the case of CDS, it is an illiquidity premium instead of an illiquidity discount.

**Definition 3.** The illiquidity premium, \( d_c \), in the CDS price is defined as the difference between the frictionless CDS price and the price with search frictions:

\[
d_c \equiv p_c - \lim_{\lambda_c, \lambda_b \to \infty} p_c = x_{ch} + x_{cl} - 2y - \frac{1 - \phi}{\phi} (r + \gamma_u + q_{cs}) \Delta_{hoc}
\]

**Proposition 9.** An exogenous increase in CDS market liquidity (that is, an increase in the matching efficiency of the CDS market, \( \lambda_c \)) has the following effects:

1. Bond market liquidity decreases: the illiquidity discount \( (d) \) increases and the volume of trade \( (\lambda_b \mu_{hn} \mu_{aob}) \) decreases.
2. CDS market liquidity increases: the illiquidity premium \( (d_c) \) decreases and the volume of trade \( (\lambda_c \mu_{hn} (\mu_{in} + \mu_{aoc})) \) increases.
3. The bond price \( (p_b) \) decreases.
4. The CDS price \( (p_c) \) decreases.
Results 9.1 and 9.2 show that an exogenous increase in CDS market liquidity pushes bond and CDS market liquidity in opposite directions. The CDS market had a positive externality on bond market liquidity in the presence of search frictions in the CDS market. Lower search frictions in the CDS market reverses this positive externality.

Due to these opposite movements in bond and CDS market liquidity, CDS and bond prices also change in opposite directions in response to an exogenous increase in CDS market liquidity. The CDS price becomes cheaper (which in the data may be perceived as a decrease in credit risk) but bond yields increase.

Generally, it is not obvious whether CDS market liquidity should increase or decrease the CDS price, and hence whether it is the buyer or the seller of a CDS contract that extracts the rent. The empirical evidence is also mixed. Thus, the model predictions clarify this ambiguity. We learn that it is the CDS seller that captures the spread.

**Proposition 10.** An exogenous increase in bond market liquidity (that is, an increase in the matching efficiency of the bond market, $\lambda_b$) has the following effects:

1. Bond market liquidity increases: the illiquidity discount ($d$) decreases and volume of trade ($\lambda_b \mu_{hn} \mu_{aob}$) increases.
2. CDS market liquidity decreases: the illiquidity premium ($d_c$) increases and the volume of transactions ($\lambda_c \mu_{hn} (\mu_{ln} + \mu_{aoc})$) decreases.
3. The bond price ($p_b$) increases.
4. The price of a CDS contract ($p_c$) increases.

Proposition 10 shows that an exogenous increase in bond market liquidity has exactly the opposite effects from an exogenous increase in CDS market liquidity.

### 3.1 Implications on CDS-bond basis

The CDS-bond basis is the CDS spread (the price of a CDS contract) minus bond yields after adjusting for the risk-free rate. It captures the difference between how bond and CDS markets price the same underlying credit risk. In a frictionless world, the difference and hence the basis should be zero. A large body of empirical studies document a persistent deviation of the basis from zero. The recent literature on CDS, as a result, studies the determinants of the CDS-bond basis such as the relative liquidity of bond and CDS markets. Empirical analysis is, however, confounded by endogeneity issues as asset prices and liquidity are interdependent within as well as across markets.

Using the model, we can tease out cleanly how exogenous variations in bond and CDS market liquidity affect the CDS-bond basis. Propositions 9 and 10 showed how exogenous variations in bond and CDS market liquidity change the endogenous liquidity of both markets, and consequently prices of both assets. These effects on prices directly translate into changes in the CDS-bond basis which I highlight next.
Corollary 1. The effect of exogenous variations in CDS and bond market liquidity on the CDS-bond basis.

1. An exogenous increase in bond market liquidity increases the CDS-bond basis (the basis becomes more positive).

2. An exogenous increase in CDS market liquidity decreases the CDS-bond basis (the basis becomes more negative).

An exogenous increase in bond liquidity increased bond prices while decreasing CDS market liquidity. This means CDS spreads (that is, premiums) increase while bond yields decrease, and hence the increase in the basis. This is consistent with Bai and Collin-Dufresne (2011) that find that bond market liquidity increases the basis (in particular, they find bond illiquidity measured by bid-ask spreads decreases the basis). However, they do not control for CDS market liquidity. Aree, Mayordomo, and Peña (2012) also find that bond market liquidity (relative to CDS market liquidity) increases the basis.

An exogenous increase in CDS market liquidity - through its effect on liquidity of both markets - decreases CDS spreads while increasing bond yields. As a result, the basis declines (part of 2 Corollary 1).

4 Discussion

4.1 Extensions Outside the Scope of the Paper

In this section, I discuss possible extensions of the paper and how they might affect the main results of the paper.

An interesting extension would be to introduce aggregate shocks and analyze which market responds faster and more strongly to the shock. Hence, we can study the price discovery mechanism and how it is affected by exogenous variations in, for example, liquidity of the two markets.

In the model, as is standard in the search literature of financial assets, I assumed that agents can hold either 0 or 1 unit of the bond or a CDS contract. An interesting extension would be allow for more than one unit of a CDS position. This would help us contrast the derivative versus fixed supply features of CDS and bonds, respectively. Such an extension would most likely make the complementary effect (that is, the positive externality effect of CDS) even stronger. It would also allow us to contrast how shorting credit risk using the CDS market is different from shorting bonds directly. Directly shorting bonds is limited by the supply of bonds while similar constraints do not exist for CDS contracts.

To that end, another possible extension is to allow for bond shorting. CDS contracts exist in the model because I assumed that it is cheaper to short credit risk using the CDS market than directly shorting bonds. To isolate the marginal effect of allowing naked CDS buyers, I compare the environment with naked CDS buyers to a benchmark environment without naked CDS buyers. Allowing bond shorting would just change the benchmark environment. Whether the benchmark environment includes bond shorting or not, the marginal effect of naked CDS buyers should be the same.
I assumed that buyers of a CDS contract are naked buyers only and not buying CDS to hedge their bond exposures. Allowing for “covered” CDS buyers is a possible extension. The main results are not likely to change because we are still interested in the marginal effect of naked buyers relative to some benchmark whether the benchmark includes covered CDS buyers or not. The effect of allowing bond holders to buy CDS is more obvious (it should increase bond market liquidity) and hence in of itself not interesting.

Another possibility is to endogenize the search intensity \((\lambda_b, \lambda_c)\) and analyze how traders adjust their search intensity into bond versus CDS markets when the CDS market is introduced. Typically, when the search intensity is endogenous in a setting with multiple but identical assets, a multiple equilibrium can arise. As long as entry is endogenous, in the equilibrium where both markets exist (which is what we see in reality), there should still be an increase in the number of long traders due to naked CDS buyers. If traders are heterogenous in some dimension, typically, there can be some form of a clientele equilibrium where one type of agents trade in one market and the other types trade in the other.

### 4.2 The Importance of Naked Purchases of CDS

In the data it is impossible to measure what proportion of CDS purchases are naked versus covered since we do not see bond holdings of CDS buyers. Even bonds outstanding relative to CDS outstanding is not a good proxy for covered purchases since we cannot tell that it is the bondholders purchasing the CDS contracts. Conversations with traders reveal that they generally use CDS to trade on the overall credit risk of the reference entity, and use bonds to trade on bond-specific credit risk. Such use of a CDS would be a naked purchase. CDS outstanding on European sovereign single-name reference entities have largely disappeared since the permanent naked CDS ban. These observations suggest that naked CDS purchases possibly constituted a large proportion of the market.

### 4.3 Search vs. Asymmetric Information

Another plausible effect of the CDS market is that as an instrument to trade on negative news, shorting credit risk through the CDS market may amplify a potential “run” on sovereign bond markets which then leads to a further liquidity dry-up in the bond market. This is an informational story. Although plausible, this mechanism on its own cannot explain why different bans would affect bond market liquidity differently.

In the above scenario, informational asymmetry was between the sovereign and bond investors as a group. Illiquidity could also arise from asymmetric information amongst traders as in [Kyle (1985)](Kyle1985) and [Glosten and Milgrom (1985)](GlostenMilgrom1985). The search framework is better suited for sovereign bond markets for two reasons. First, illiquidity in sovereign bond markets is more due to fragmentation of trades across heterogenous bonds. Second, asymmetric information and insider trading is less severe with respect to governments than with respect to individual firms.
5 Institutional Details

This section describes sovereign bond and CDS markets and the European regulations that banned naked purchases of CDS as discussed in Sambalaibat (2014). For more details, see Sambalaibat (2014).

5.1 Sovereign Bond Market

Generally, government bonds trade in over-the-counter markets. A trader in the U.S., for example, shops for sovereign bonds using phone calls, emails, messages and quotes through Bloomberg. Locating a particular bond issue can be at times impossible. Bond markets of few governments, however, are organized as electronic markets. The U.S. Treasury market, for example, is organized as an electronic limit order market. Most of the Italian government bonds trade on an inter-dealer trading platform, called MTS, that functions similar to an electronic limit order market and is not accessible to individual investors. Despite the fact that the MTS is organized similar to an equity market and is one of the largest and most liquid government bond markets, trade is fragmented across heterogenous bonds and liquidity per bond is low. According to Pelizzon, Subrahmanyam, Tomio, and Uno (2013), daily trading volume and the number of trades per bond on the MTS are comparable to the US municipal bond and the US corporate bond markets.

5.2 CDS Market

As discussed before, credit default swaps are over-the-counter derivative contracts that resemble insurance protection against a default or a similar event (referred to as a “credit event”) on bonds of a firm or a government (the “reference entity”). A buyer of a CDS protection pays a periodic fee (equivalently, the CDS price, premium, or spread) until either the contract matures or a credit event occurs. In return, the seller pays the buyer the protection amount that was purchased (called “notional”) in the event of default (or a similar event) on any one of the bonds of the reference entity that is covered by the CDS contract. CDS contracts are therefore written on firms and governments as a whole and not at a level of individual bond issues.

CDS contracts specify the reference entity, the contract maturity, the notional amount, the set of bonds of the reference entity that the contract covers, and the default events that constitute a credit event. The standard notional amounts are in the range of $10-20 million. Prices of CDS contracts

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17 See Cheung, Rindi, and De Jong (2005), Dufour and Skinner (2004), and Pelizzon, Subrahmanyam, Tomio, and Uno (2013) for more information on MTS trading platforms.

18 For example, suppose you are a holder of bond “A” of the Greek government and Greece defaults on another bond “B.” If both bonds are covered by the contract, you will be still be paid out even if your bond “A” has not been defaulted on.

19 This is comparable to the most common transaction sizes of 5, 10, 25 million euros in, for example, the MTS Global Market (see Cheung, Rindi, and De Jong (2005)).
are paid quarterly and are quoted as annualized percentages of the contract notional.\textsuperscript{20}

The governing body for the CDS market, the International Swaps and Derivatives Association (ISDA), determines whether a credit event has occurred or not.\textsuperscript{21} The standard credit events for sovereign CDS are Failure to Pay and Debt Restructuring.\textsuperscript{22} Protection buyers get paid the difference between the notional and the recovery value that is determined through a special post-credit-event auction. For example, if an investor bought a CDS contract with a notional of $10 million and the recovery rate is 25\%, she receives $7.5 million in cash. The ISDA finalizes the actual list of eligible bonds that can be submitted into the auction and oversees the auction. At the end of the auction, all bonds submitted into the auction are bought and sold at the same final bond price, and this final price is the price or the recovery rate that settles all CDS contracts on that reference entity. The recovery value is effectively the price of the defaulted bonds. Although cash settlements have become standard now, CDS buyers also have the option of physically settling their contracts by selling their bonds during the auction.

5.3 The Permanent CDS Ban

Throughout 2011, market participants faced uncertainty over whether the EU would adopt measures to ban naked CDS. The uncertainty was finally resolved on October 18, 2011 when, after months of negotiations, the European Parliament and the EU states passed a law to permanently ban naked CDS. The legislation applied to all CDS transactions referencing governments of the EU regardless of the geographic location of the transaction or the legal jurisdiction of the financial institution involved in the transaction.

The final draft of the law was published March 2012 (Regulation EU No 236/2012). Although the legislation was to be in effect beginning November 1, 2012, the March 2012 regulation stated that traders who enter new contracts after March 2012 would have to unwind them by November 2012.

\textsuperscript{20}For example, if the price of a CDS contract with $10 million notional is 200 basis points, the protection buyer pays $0.2 million annually in quarterly installments of $0.05 million. The price of a CDS contract can be thought of as, in its simplest form, the probability of default times one minus the recovery rate. For example, if a one year CDS contract is trading at 200 basis points, and the recovery rate was zero, then the implied probability of default is 2\%.

\textsuperscript{21}Credit events are decided by the “determination committee” of the ISDA which consists of 10 big dealer banks (e.g. Bank of America, Barclays, BNP Paribas, Citibank, Credit Suisse, Deutsche Bank, Goldman Sachs, JPMorgan, Morgan Stanley, and UBS) and five buy side firms that tend to be hedge funds.

\textsuperscript{22}For corporate CDS, bankruptcy is an additional standard credit event. There are three kinds of restructuring that vary by how restrictively they limit the set of eligible bonds: Modified Restructuring (MR), Modified Modified Restructuring (MMR), and Complete (or “old”, “full”) Restructuring (CR). MR is the most restrictive limiting eligible bonds to have maturity of up to 30 month after the declaration of a credit event, then MMR with 60 month maturity, and CR is the least restrictive with the standard 30-year maturity limit on bonds. CDS on North American reference entities usually feature MR (except CDS on high credit risk firms tend to completely exclude any debt restructuring as a credit event), while CDS on European firms feature the less restrictive MMR. Debt restructuring on CDS on sovereigns, on the other hand, most commonly specify CR.
Contracts entered into before March 2012 could remain in place even beyond November 2012. Figure 10 compares the total CDS purchased referencing EU governments versus countries not affected by the ban. We see that the total amount of CDS purchased on EU sovereigns started to dramatically decrease starting around the time that the law was passed and has been declining ever since. This decrease did not occur for countries not affected by the ban. Thus, anticipating the difficulty of renewing contracts beyond March 2012, traders started to decrease their activity already beginning the fall of 2011.

A CDS purchase was considered covered if it was hedging a portfolio of assets that was correlated with government bonds of the reference entity. In particular, the value of the portfolio had to have a historical correlation of at least 70% with the government bond price over a period of at least 12 months prior to the CDS purchase. If a CDS purchase could not satisfy this at the time of the purchase, it would be considered naked and hence prohibited. The underlying portfolio could consist of, for example, long positions in private entities within the reference entity country or even long positions through CDS itself. The correlation requirement would be automatically satisfied if the underlying position being hedged consisted of governments bonds (at federal and local levels of the government), the liability of state enterprises, and the liability of enterprises guaranteed by the sovereign. The legislation exempted market making activities.

After the purchase, traders did not have to maintain the correlation throughout the CDS contract to allow for prices of the underlying assets to vary. But the size of the underlying positions had to remain “proportional” to the amount of CDS purchased. In other words, a trader could not buy bonds with the intent of selling them back once she purchases CDS. The regulation was enforced by putting the responsibility on institutions to keep track of their positions. Upon request, institutions were supposed to be able to prove that their CDS purchases for the purpose of hedging.

Figure 10 in the Appendix shows the time series of the total amount of CDS purchased around the ban. Between the EU’s introduction of the permanent ban in October 2011 and the end of my sample period in June 2013, the total amount of CDS purchased referencing governments of the EU declined by one third, while a similar decline did not occur for countries not affected by the ban (in dashed line). Today, trading in European sovereign single-name contracts has essentially dried up according to ISDA (2014). These observations suggest that naked CDS positions played an important role in the CDS market and possibly constituted a large proportion of the total CDS outstanding.

Figure 11 plots the cross country average of the bond bid-ask spread for the groups of countries (affected versus unaffected). After the regulation was introduced, the countries affected by the ban observed a widening of the bond bid-ask spread compared to countries not affected by the ban. This suggests the ban had a detrimental effect on liquidity of the underlying bonds.

Market participants were generally confused about how to actually interpret and satisfy the restrictions of the regulation.
5.4 The Temporary CDS Ban

On Tuesday May 18th 2010, Germany prohibited naked purchases of CDS referencing Eurozone governments. As recent as month prior to the ban Germany’s rhetoric had been that there is no need to ban naked CDS trading. The regulation was unexpected by market participants and was implemented within the same day that the media first reported it. News about the ban first appeared around 1pm on Tuesday May 18, 2010 on Reuters. But the official details of the legislation did not emerge until late in the evening around 9:30pm. The regulation was effective from midnight the same day (within two and half hours from the release of the official statement) and was to be in effect through March 31, 2011. However, later on July 27, 2010 the regulation was made permanent.

The regulation also banned the naked short selling of 10 leading German financial stocks and the naked short selling of Eurozone governments bonds that were allowed to be listed on Germany’s domestic stock exchange. The naked bond short selling restriction, as a result, applied to only a few German and Austrian bonds.

The May 18th 2010 regulation did not specify the territorial scope of the regulation. So it is not clear whether market participants interpreted the regulation to apply to all transactions regardless of the geographic location and the institution. However, according to Allen & Overy LLP and ISDA’s conversations with BaFin (Germany’s financial regulatory body), BaFin confirmed that the regulation applied to transactions where at least one of the counterparties is located in Germany. It would not, for example, apply to a transaction between the New York branch and the London branch of Deutsche Bank.

Figure 12 plots the cross country average of the bond bid-ask spread. The dashed line shows the average for the EU countries that were not affected by the ban (i.e. naked CDS referencing these countries could still be purchased), while the solid line plots the average for the EU countries affected by the ban (i.e. the Eurozone countries). Two vertical lines are drawn for the week before the ban and the week of the ban. We see that for the countries affected by the ban, there was a large and sudden narrowing of the bond bid-ask spread, while this did not occur for the countries not affected by the ban. Figure 13 in the Appendix demonstrates the time series of CDS net notional around the ban.

6 Conclusion

This paper builds a search-theoretic model of over-the-counter bond and CDS markets that features an endogenous liquidity interaction between the two markets and endogenous funding liquidity.

My model shows that, in the long run, trading activity and liquidity in the CDS market spills over to the bond market and increases bond market liquidity because there is an increasing returns to scale to searching simultaneously in multiple substitute markets. This spillover effect arises from endogenizing entry and consequently the aggregate number of investors which, in stan-
standard search models, is kept fixed. In the short run, however, the entry rate is sticky and is unaffected by the CDS market and, as a result, introducing the CDS market decreases liquidity in the bond market.

These effects are economically relevant. Sambalaibat (2014) documents that different naked CDS bans implemented in Europe (one permanent and the other temporary) had completely opposite effects on bond market liquidity. The implications from the long-run versus the short-run effects in the model help rationalize the observed changes in bond market liquidity after these CDS bans.
A Appendix: Proofs

Agents’ flow value equations are analogously derived to (6):

\[ rV_{in} = \gamma_u (0 - V_{in}) + q_{cs} (V_{isc} - V_{in}) \]  
(14)
\[ rV_{hob} = \delta_b + x_b - y + \gamma_d (V_{aob} - V_{hob}) \]  
(15)
\[ rV_{aob} = \delta_b + y + q_{bb} (0 - V_{aob} + p_b) \]  
(16)
\[ rV_{hoc} = \rho_c - (\delta_c - x_{ch}) - y + \gamma_d (V_{aoc} - V_{hoc}) + \gamma_u (V_{hn} - V_{hoc}) \]  
(17)
\[ rV_{aoc} = \rho_c - \delta_c - y + q_{cs} (0 - V_{aoc}) + \gamma_u (0 - V_{aoc}) \]  
(18)
\[ rV_{isc} = -p_c + (\delta_c + x_{cl}) - y + \gamma_u (0 - V_{isc}) \]  
(19)

Proof of Proposition [7]. The proof of uniqueness is shown in Lemma [1] and the proof of existence is shown in Lemma [2].

Lemma 1. Suppose (7) holds, then the steady state equilibrium is unique.

Proof. First fix \( \rho \), then using the in-flow out-flow equations and the market clearing conditions (2)-(4), \( \mu_{in}, \mu_{hob}, \mu_{aob}, \mu_{hoc}, \mu_{aoc}, \mu_{isc} \) can be solved as a function of \( \mu_{hn} \):

\[ \mu_{in} = \frac{F_i}{\gamma_u + \lambda_c \mu_{hn}} \]  
(20)
\[ \mu_{hob} = \frac{S \lambda_b \mu_{hn}}{\lambda_b \mu_{hn} + \gamma_d} \]  
(21)
\[ \mu_{aob} = \frac{S}{\lambda_b \mu_{hn}} - \frac{S \lambda_b \mu_{hn}}{\lambda_b \mu_{hn} + \gamma_d} \]  
(22)
\[ \mu_{hoc} = \frac{\gamma_u (\lambda_c \mu_{hn} + \gamma_d + \gamma_u)}{\gamma_d F_i \lambda_c \mu_{hn}} \]  
(23)
\[ \mu_{aoc} = \frac{\gamma_u (\lambda_c \mu_{hn} + \gamma_u) (\lambda_c \mu_{hn} + \gamma_d + \gamma_u)}{\lambda_c F_i \mu_{hn}} \]  
(24)
\[ \mu_{isc} = \frac{\gamma_u (\lambda_c \mu_{hn} + \gamma_u)}{\gamma_u (\lambda_c \mu_{hn} + \gamma_u)} \]  
(25)

And \( \mu_{hn} \) itself is a solution to:

\[ (1 + \rho) V_{hn} - \gamma_d \mu_{hn} \left( \frac{S \lambda_b}{\lambda_b \mu_{hn} + \gamma_d} + \frac{\lambda_c F_i}{\gamma_u (\lambda_c \mu_{hn} + \gamma_d + \gamma_u)} + 1 \right) = 0 \]  
(26)

The LHS of (26) is positive at \( \mu_{hn} = 0 \), decreasing in \( \mu_{hn} \), and is negative for large \( \mu_{hn} \), hence (26) has a unique positive solution. Thus, (26) uniquely determines \( \mu_{hn} \) and has a positive solution, while other \( \mu \)'s are uniquely determined by (20)-(24). Next, once \( \mu \)'s are solved, the value functions and prices are uniquely determined by a linear system of equations: (6), (14)-(19), and (21)-(25).

We are left with the endogenous entry decisions:

\[ \rho = \begin{cases} 1 & V_{hn} (\rho) > O_h \\ [0, 1] & V_{hn} (\rho) = O_h \\ 0 & V_{hn} (\rho) < O_h \end{cases} \]  
(27)

There are three cases: two corner solutions \( \rho = 0 \), and \( \rho = 1 \), and an interior solution. Next, I show that \( V_{hn} \) is strictly decreasing in \( \rho \), which will imply that under each case the equilibrium is unique. The derivation in the proof of existence shows that:

\[ V_{hn} = \frac{q_{bs} x_b \phi + \Delta_{hoc} q_{ch} (r + \gamma_d + q_{bb} (1 - \phi))}{(r + \gamma_d) k} \]
where
\[
\Delta_{hoc} = x_{ch} + \left(q_{cs} + r + \gamma_d + \gamma_d\right) \frac{x_{cl} - 2y}{r + \gamma_d + q_{cs}} - \frac{1}{k} q_{bs} \phi_x
\]

None of the \(\mu\)'s other than \(\mu_{hn}\) directly depend on \(\rho\) but depend only indirectly through \(\mu_{hn}\), thus we write:

\[
\frac{\partial V_{hn}(\rho)}{\partial \rho} = \frac{\partial \mu_{hn}}{\partial \rho} \left( \frac{\partial V_{hn}}{\partial q_{bs}} \frac{\partial q_{bs}}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cb}} \frac{\partial q_{cb}}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cs}} \frac{\partial q_{cs}}{\partial \mu_{hn}} \right) + \partial V_{hn} \frac{\partial q_{cs}}{\partial \mu_{hn}}
\]

Next, I derive \(\partial V_{hn}/\partial q_{bs}\), \(\partial V_{hn}/\partial q_{cb}\), \(\partial V_{hn}/\partial q_{cs}\), and \(\partial V_{hn}/\partial q_{cs}\):

\[
\frac{\partial V_{hn}}{\partial q_{bs}} = \phi_h (r + \gamma_d + q_{bs} \phi_t) B
\]
\[
\frac{\partial V_{hn}}{\partial q_{cb}} = \phi_h (r + \gamma_d + q_{cb} \phi_t) \frac{C}{k (r + \gamma_d) C (30)}
\]
\[
\frac{\partial V_{hn}}{\partial q_{cs}} = \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t)}{k (r + \gamma_d) C} \left( \frac{\phi_t A}{\phi_h C} - \frac{(r + \gamma_d + q_{cs} \phi_t) C}{(q_{cs} + r + \gamma_u)^2} \right)
\]

where
\[
B \equiv x_b + \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t) A}{k} - A - \frac{(r + \gamma_d + q_{cb} \phi_t) x_b}{k}
\]
\[
A \equiv x_ch + \frac{x_{cl} - 2y}{r + \gamma_d + \gamma_u} q_{cs} + \frac{q_{bs} \phi_x}{r + \gamma_d + q_{bs} \phi_h + q_{bs} \phi_t}
\]
\[
C \equiv r + \gamma_d + q_{cs} \phi_t + \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t)}{k}
\]

From here, \(\partial V_{hn}/\partial q_{cb} > 0\) while \(\partial (\lambda_c (\mu_{aoc} + \mu_{hn})) / \partial q_{cs} < 0\) implying that the third term in (A.28) is negative. Since \(\partial V_{hn}/\partial q_{cs} < 0\), the fourth term (A.28) is also negative. But the sign of both \(\partial V_{hn}/\partial q_{bs}\) and \(\partial V_{hn}/\partial q_{cs}\) depend on the sign of \(B\). Thus, consider \(B\):

\[
B = x_b + \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t)}{k} q_{cb} \frac{A}{C} - \frac{q_{cb} A}{C} - \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t)}{k} x_b
\]
\[
= x_b \left( 1 - \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t)}{k} \right) - \left( 1 - \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t)}{k} \right) \frac{q_{cb} A}{C}
\]
\[
= \left( 1 - \frac{q_{cb} (r + \gamma_d + q_{cb} \phi_t)}{k} \right) \left( x_b - \frac{q_{cb} A}{C} \right)
\]

First, \(0 < \frac{q_{bs} \phi_x + \gamma_d r}{k} < 1\) and \(0 < \frac{q_{cb}}{k} < 1\). To see the latter, let \(\phi_t = \phi_h\), then \(C > q_{cs}\). From Assumption 2:

\[
\mu_{hn} + \mu_{hoc} + \mu_{hoc} = \frac{F_h}{\gamma_d} > S + \frac{F_i}{\gamma_u}
\]

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But using the CDS market clearing condition, we have \( \frac{F_l}{\gamma_a} = \mu_l + \mu_{lsc} = \mu_l + (\mu_{hoc} + \mu_{aoc}) \). Thus,
\[
\mu_{hn} + \mu_{hoc} + \mu_{hob} > S + \mu_l + (\mu_{hoc} + \mu_{aoc})
\]
Cancel \( \mu_{hoc} \).
\[
\mu_{hn} + \mu_{hob} > S + \mu_l + \mu_{aoc}
\]
\[
\mu_{hn} > (S - \mu_{hob}) + \mu_l + \mu_{aoc} > \mu_l + \mu_{aoc}
\]
Hence, \( q_{cs} > q_{cb} \) and \( C > q_{cs} > q_{cb} \). Thus, the term in the first bracket of \( B \) is positive. Now consider the term in the second bracket of \( B \), \( x_b - q_{cb} \frac{A}{x_b} = x_b - q_{cb} \Delta_{hoc} \):
\[
\begin{align*}
x_b - q_{cb} \Delta_{hoc} &= x_b - q_{cb} \left( x_{ch} + \frac{(x_{cl} - 2y)(q_{cs} + r + \gamma_u + \gamma_d)}{q_{ch} + q_{cb} + q_{cs} + r + \gamma_u + \gamma_d} - \frac{q_{cb} \phi_b}{k} x_b \right) \\
&= x_b - \frac{x_{ch} + (x_{cl} - 2y) \left( 1 + \frac{\gamma_d}{q_{ch} + q_{cb} + q_{cs} + r + \gamma_u + \gamma_d} \right) - \frac{q_{cb} \phi_b}{k} x_b}{\frac{r + \gamma_d + \gamma_u + q_{cb} \phi_b}{q_{ch} \phi_b} + \frac{k - q_{cb} \phi_b}{k}} \\
&= \left( \frac{r + \gamma_d + \gamma_u + q_{cs} \phi_l + q_{cb} \phi_b}{q_{cb} \phi_b} \right) x_b - \left( x_{ch} + (x_{cl} - 2y) \left( \frac{q_{cs} + r + \gamma_u + \gamma_d}{q_{cs} + r + \gamma_u} \right) \right)
\end{align*}
\]
The sign of the expression depends on the numerator:
\[
\left( \frac{r + \gamma_d + \gamma_u + q_{cs} \phi_l + q_{cb} \phi_b}{q_{cb} \phi_b} \right) x_b - \left( x_{ch} + (x_{cl} - 2y) \left( \frac{q_{cs} + r + \gamma_u + \gamma_d}{q_{cs} + r + \gamma_u} \right) \right)
\]
This expression is positive from \([7]\). Thus, \( \frac{\partial \mu_{hn}}{\partial \rho} > 0 \) and together with \( \frac{\partial \mu_{hoc}}{\partial \mu_{aoc}} < 0 \) implies that the first term of \([A.28]\) is negative. Also, since \( \frac{\partial \mu_{hn}}{\partial \mu_{aoc}} < 0 \), the second term of \([A.28]\) is also negative.

Finally from \([A.26]\) and using the Implicit Function Theorem,
\[
\frac{\partial \mu_{hn}}{\partial \rho} = \frac{F_h}{\gamma_d \left( \frac{\lambda_\gamma \gamma_d}{(\lambda_\gamma \mu_{hn} + \gamma_d)^2} + \frac{\lambda_\gamma f(\gamma_u + \gamma_d)}{\gamma_u (\lambda_\gamma \mu_{hn} + \gamma_u)^2} + 1 \right)}
\]
Thus, \( \frac{\partial \mu_{hn}}{\partial \rho} > 0 \), and, consequently, \( \frac{\partial V_{hn}(\rho)}{\partial \rho} < 0 \). \( \square \)

**Lemma 2. Existence**

**Proof.** To show existence we verify that the conjectured optimal trading strategies are in fact optimal. In particular, first, we show that the total surplus from trading the bond is positive: \( \omega_b = V_{hob} - V_{hn} - V_{aob} > 0 \). By construction, this will ensure that individual surpluses to the buyer and the seller of the bond are positive: a high type agent optimally chooses to buy the bond, and an average type agent prefers to sell her bond. Second, we show that the total surplus from trading CDS is positive \( \omega_c = V_{hoc} - V_{hln} + V_{lsc} - V_{ln} > 0 \). This will imply that the high type agents will want to sell CDS, while low type agents will want to buy CDS. Third, we verify that the average type agents will prefer quit being a CDS seller: \( 0 - V_{aoc} > 0 \). Thus, agents who have previously sold CDS when they were high types will prefer to find another seller to take over their side of the trade and exit the market with zero utility. I proceed by first deriving \( \omega_b, \omega_c, V_{aoc} \).

Subtracting \( rV_{ln} \) \([A.14]\) from \( rV_{lsc} \) \([A.19]\) and defining \( \Delta_{lsc} \equiv V_{lsc} - V_{ln} \), we get:
\[
\Delta_{lsc} = \frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}}
\]

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From (3):
\[ \Delta_{hoc} = \frac{\phi}{1 - \phi} \Delta_{isc} = \frac{\phi}{1 - \phi} \frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}} \]

Also from the value function of \( V_{aoc} \),
\[ V_{aoc} = \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cbs}} \quad \text{(A.29)} \]

Using (A.17) and substituting in the expression for \( V_{aoc} \):
\[ rV_{hoc} = p_c - (\delta_c - x_{ch}) - y + \gamma_d \left( \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cbs}} - V_{hoc} \right) - \gamma_u \Delta_{hoc} \]

Add \( \gamma_d V_{hoc} \) to both sides:
\[ (r + \gamma_d)V_{hoc} = p_c - (\delta_c - x_{ch}) - y + \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cbs}} - \gamma_u \Delta_{hoc} \]

Subtract \( (r + \gamma_d)V_{hn} \) from both sides:
\[ (r + \gamma_d + \gamma_u)\Delta_{hoc} = p_c - (\delta_c - x_{ch}) - y + \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cbs}} - (r + \gamma_d)V_{hn} \]

Thus, we have three equations and three unknowns, \( \Delta_{hoc}, p_c, V_{hn} \):
\[ \Delta_{hoc} = \frac{\phi}{1 - \phi} \frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}} \]
\[ (r + \gamma_d + \gamma_u)\Delta_{hoc} = p_c - (\delta_c - x_{ch}) - y + \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cbs}} - (r + \gamma_d)V_{hn} \]
\[ V_{hn} = \frac{q_{bcb} + \Delta_{hoc}q_{cb}(r + \gamma_d + q_{cb}(1 - \phi))}{(r + \gamma_d)k}, \quad \text{(A.30)} \]

where the latter comes from the solution to the equations for \( V_{hoc}, V_{aob}, \) and \( V_{hn} \). The solution for \( \Delta_{hoc} \) is given by:
\[ \Delta_{hoc} = \frac{x_{ch} + (q_{cbs} + r + \gamma_u + \gamma_d) \frac{x_{cl} - 2y}{r + \gamma_u + q_{cbs}} - \frac{1}{r} q_{bcb} x_b}{(r - r_{q_u + r + \gamma_u + \gamma_d}) + \frac{1}{r} q_{bcb}(r + \gamma_d + (1 - \phi) q_{cb})} \quad \text{(A.31)} \]

From here:
\[ p_c = \delta_c + x_{cl} - y - \frac{1 - \phi}{\phi} (r + \gamma_u + q_{cbs}) \Delta_{hoc} \quad \text{(A.32)} \]
\[ \omega_c = \frac{1}{\phi} \Delta_{hoc} \]

Using the solution to the equations for \( V_{hoc}, V_{aob}, \) and \( V_{hn} \):
\[ \omega_b = \frac{x_b}{k} - \frac{q_{cbs} \Delta_{hoc}}{k} \quad \text{(A.33)} \]

To consider small search frictions, define \( \epsilon \equiv \frac{1}{\lambda_b} \) and \( n \equiv \frac{\lambda_b}{\lambda_b} \). We show existence for \( \epsilon = 0 \). Then by continuity, existence is established in the neighborhood of \( \epsilon \equiv 0 \) or for small search frictions. With the change of variables, (A.26) becomes:
\[ (1 + \rho)F_b - \gamma_d \mu_{hn} \left( \frac{S}{\mu_{hn} + \epsilon \gamma_d} + \frac{n F_1}{\gamma_u (n \mu_{hn} + \epsilon (\gamma_d + \gamma_u)) + 1} \right) = 0 \quad \text{(A.34)} \]
From (A.34), for any $\rho \in [0,1]$, $\mu_{hn}$ asymptotically converges to $\mu_{hn} = \frac{(1+\rho)F_h}{S} - (S + \frac{F_h}{S})$ therefore $0 < \lim_{\lambda_b,\lambda_c \to \infty} \mu_{hn} < \infty$ and $\lim_{\lambda_b,\lambda_c \to \infty} q_{bs} = \infty$. This also implies from (A.22) that $\lim_{\lambda_b,\lambda_c \to \infty} \mu_{aob} = 0$ and $q_{bs}$ converges to a finite number. Analogously, $\lim_{\lambda_b,\lambda_c \to \infty} q_{cs} = \infty$ and from (A.20) and (A.24):

$0 < \lim_{\lambda_b,\lambda_c \to \infty} q_{cb} < \infty$.

To show $\omega_c > 0$ using these limits, consider the numerator of $\Delta_{hoc}$:

$$\Delta_{hoc} = x_{cb} + (q_{cs} + r + \gamma_u + \gamma_d) \frac{x_{cl} - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bs} \phi x_b$$

Using the above limits of $q_{cs}$, $q_{bs}$, and $q_{bb}$, it converges to $x_{cb} + x_{cl} - 2y$ which is positive by Assumption 1.

From (A.29), in order for $V_{aoc} < 0$, the CDS price has to be such that $p_c < \delta_c + y$. From (A.31) and (A.32):

$$p_c = (\delta_c + x_c) - \gamma_u \left( 1 - \phi_h \right) \left( \frac{q_{cs} + r + \gamma_u}{r + \gamma_u + q_{cs}} \left( \frac{x_{cb} + (x_{cl} - 2y)(q_{cs} + \gamma_d + r + \gamma_u)}{q_{cs} + r + \gamma_u} - \frac{x_{cb} \phi x_b}{q_{bs} \phi x_b} \right) \right) \phi_h \left( \frac{q_{cs} + r + \gamma_u}{q_{cs} + r + \gamma_u} \right)$$

This converges to $\delta_c + y - x_{cb}$, which is less than $\delta_c + y$. Thus, $V_{aoc} < 0$. Average types will not want to buy CDS because the flow utility would be $\delta_c - y - p_c$. Given that $p_c \to \delta_c + y - x_{cb}$, the flow utility $\delta_c - y - p_c$ converges to $x_{cb} - 2y$ which is negative by Assumption 1. To show $\omega_b > 0$, consider the numerator of (A.33): $x_b - q_{cb} \Delta_{hoc}$. Since $0 < \lim_{q_{cb} < \infty}$ and $\Delta_{hoc}$ converges to zero, $x_b - q_{cb} \Delta_{hoc}$ converges to $x_b > 0$. The above results show existence for $\epsilon = 0$. By continuity, existence is also established near $\epsilon = 0$.

**Proof of Proposition 2.** The bond price is $p_b = \phi(V_{hob} - V_{hn}) + (1 - \phi)V_{aob}$. Solving $V_{hob}$ and $V_{aob}$:

$$V_{hob} = \frac{\delta_b + x_b - y}{r} - \gamma_d \left( \frac{x_b + q_{bb} \left( 1 - \phi \right) V_{hn}}{r + \gamma_d + q_{bb} \left( 1 - \phi \right) V_{hn}} \right)$$  \hspace{1cm} (A.35)

$$V_{aob} = \frac{\delta_b + x_b - y}{r} - \frac{(r + \gamma_d) \left( \frac{x_b + q_{bb} \left( 1 - \phi \right) V_{hn}}{r + \gamma_d + q_{bb} \left( 1 - \phi \right) V_{hn}} \right)}{r + \gamma_d + q_{bb} \left( 1 - \phi \right)}$$  \hspace{1cm} (A.36)

where from the earlier derivation:

$$V_{hn} = \frac{q_{bb} x_b \phi + \Delta_{hoc} q_{cb} \left( r + \gamma_d + q_{bb} \left( 1 - \phi \right) \right)}{(r + \gamma_d) k}$$  \hspace{1cm} (A.37)

Thus, we derive the limits of $q$’s, and $\Delta_{hoc}$ as $\lambda_b \to \infty$ for an arbitrary $\lambda_c$. With the change of variable, $\epsilon \equiv \frac{1}{\lambda_c}$, (A.26) becomes:

$$\left( 1 + \rho \right) F_h - \gamma_d \mu_{hn} \left( \frac{S}{\mu_{hn} + \epsilon \gamma_d} + \frac{\lambda_c F_l}{\gamma_u \left( \lambda_c \mu_{hn} + \gamma_d + \gamma_u \right)} + 1 \right) = 0$$

For $\epsilon = 0$,

$$\frac{1 + \rho}{\gamma_d} F_h - S - \mu_{hn} \left( \frac{\lambda_c F_l}{\gamma_u \left( \lambda_c \mu_{hn} + \gamma_d + \gamma_u \right)} + 1 \right) = 0$$  \hspace{1cm} (A.38)

For any $\rho \in [0,1]$, the LHS of (A.38) is positive at $\mu_{hn} = 0$, decreasing in $\mu_{hn}$, and is negative for large $\mu_{hn}$. Hence, (A.38) has a positive finite solution, $0 < \lim_{\lambda_b \to \infty} \mu_{hn} < \infty$, and this implies $\lim_{\lambda_b \to \infty} q_{bs} = \infty$, and $k \to \infty$. This also implies from (A.22) that $\lim_{\lambda_b \to \infty} \mu_{aob} = 0$ and $q_{bs}$ converges to a finite number. Analogously, $\lim_{\lambda_b \to \infty} q_{cs} = \infty$ and from (A.20) and (A.24):

$0 < \lim_{\lambda_b \to \infty} q_{cb} < \infty$. 

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Then as discussed above, the numerator of $\Delta_{hoc}$ converges to a finite number, while the denominator converges to $\infty$, thus, $\Delta_{hoc} \to 0$. So $V_{hn} \to 0$, hence $V_{hob} \to \frac{x_b + y + r}{y}$, $V_{aob} \to \frac{x_b + y}{y}$ and $p_b \to \frac{y + x_b - y}{y}$.

Note that since $V_{hn} \to 0$, $\rho \to 0$. This is why the assumption that there is some proportion of high types who do not have an outside opportunity and always enter simplifies some of the proofs. Otherwise, as high types enter at a smaller and smaller rate, the steady state measure of high types can become smaller than $S + \frac{E}{\rho_{hn}}$. As a result, the marginal investor of the bond is not necessarily the high type and the frictionless price is not given by the valuation of the high types.

**Proof of Proposition 3** Combining (A.35)-(A.37) we get the bond price.

**Proof of Proposition 4**

Case 1.1. Parameter conditions such that $V_{hn}(\rho^{nocds}) = V_{hn}(\rho^{cds}) = O_h$.

Since the bond price is $p_b = \phi(V_{hob} - V_{hn}) + (1 - \phi)V_{aob}$, for an interior solution ($V_{hn}^{nocds} = V_{hn}^{cds} = O_h$) it is sufficient to show that $V_{hob}(q_{bb}^{nocds}) > V_{aob}(q_{bb}^{nocds})$ and $V_{aob}(q_{bb}^{cds}) > V_{aob}(q_{bb}^{nocds})$. From (A.35) and (A.36), the derivative with respect to $q_{bb}$:

$$
\frac{\partial V_{hob}}{\partial q_{bb}} = -\frac{\gamma_d ((r + \gamma_d) V_{hn} - x_b) (1 - \phi)}{r (r + \gamma_d + q_{bb}(1 - \phi))^2}
$$

$$
\frac{\partial V_{aob}}{\partial q_{bb}} = -\frac{(r + \gamma_d) ((r + \gamma_d V_{hn} - x_b) (1 - \phi)}{r (r + \gamma_d + q_{bb}(1 - \phi))^2}
$$

Thus, the condition for both $V_{hob}$ and $V_{aob}$ to be increasing in $q_{bb}$ at $q_{bb} = q_{bb}^{nocds}$ is: $(r + \gamma_d) V_{hn} - x_b < 0$ evaluated at $q_{bb} = q_{bb}^{nocds}$. The solution for $V_{hn}$ without the CDS market:

$$
V_{hn}^{nocds} = \frac{q_{bb} x_b \phi}{(r + \gamma_d)(r + \gamma_d + q_{bb}(1 - \phi)} \tag{A.39}
$$

Rearranging (A.39) we get:

$$(r + \gamma_d) V_{hn} = \frac{q_{bb} x_b \phi}{(r + \gamma_d + q_{bb}(1 - \phi)} x_b < x_b$$

Next, we show that $q_{bb} = \lambda_b \mu_{hn}$ increases with CDS. Consider the solution for $V_{hn}$ with CDS:

$$
V_{hn}^{cds} = \frac{x_b q_{bb}^{cds} \phi_{hn}}{k^{cds} (\gamma_d + r)} + \frac{q_{cb} \Delta_{hoc} (q_{bb}^{cds} \phi_{hn} + \gamma_d + r)}{k^{cds} (\gamma_d + r)} \tag{A.40}
$$

Compare this with (A.39). The fact that $V_{hn}^{nocds} = V_{hn}^{cds} = O_h$ and that the second term of (A.40) is asymptotically positive implies that:

$$
\frac{x_b q_{bb}^{cds} \phi_{hn}}{k^{cds} (\gamma_d + r)} < \frac{x_b q_{bb}^{cds} \phi_{hn}}{k^{cds} (\gamma_d + r)}
$$

The term $x_b q_{bb}^{cds} \phi_{hn}$ is strictly decreasing in $\mu_{hn}$. Thus, it has to be the case that $\mu_{hn}^{cds} > \mu_{hn}^{nocds}$.

Now consider the effect on the bond bid-ask spread, $\omega_b$, characterized in (A.33). The first term $x_b/k$ in (A.33) is strictly decreasing in $\mu_{hn}$, while the second term is asymptotically positive and arises due to CDS. Thus, it follows that $\omega_b^{cds} < \omega_b^{nocds}$.

Case 1.2. $V_{hn}(\rho^{nocds}) = O_h < V_{hn}(\rho^{cds})$. 35
This will be the case when \( V_{hn}(\rho^{cds} = 1) > O_h \), hence \( 1 = \rho^{cds} > \rho^{nocds} \). There are two regions to consider: 1) \( \lambda < \bar{\lambda} \) and 2) \( \lambda \geq \bar{\lambda} \), where \( \bar{\lambda} \) is defined such that \( p_b^{cds}(\lambda_{\bar{\lambda}}) = p_b^{nocds} \).

\[
p_{b}^{cds} = \frac{\delta_b + x_b - y}{r} - \left[ \gamma_d \frac{x_b}{r \phi^{cds}} \right] \frac{x_b}{r \phi^{cds}} - \frac{(q_{b}^{cds} + r) (1 - \phi)}{r \phi^{cds}} q_{b} \Delta_{hoc},
\]

(A.41)

and

\[
p_{b}^{nocds} = \frac{\delta_b + x_b - y}{r} - \left[ \gamma_d \frac{x_b}{r} + \phi \left( q_{b}^{cds} + r \right) \frac{x_b}{r} \right].
\]

(A.42)

In the region \( \lambda_c > \bar{\lambda} \), the complementarity effect (from the increased entry rate \( 1 = \rho^{cds} > \rho^{nocds} \)) still dominates the substitution effect and \( p_{b}^{cds} > p_{b}^{nocds} \) and \( \Delta^{cds} < \Delta^{nocds} \). At \( \lambda_c = \bar{\lambda} \), however, the substitution effect starts taking over the complementarity effect and as \( \lambda_c \) decreases further, in the region \( \lambda_c < \bar{\lambda} \), the substitution effect is larger. Consequently, \( p_{b}^{cds} < p_{b}^{nocds} \) and \( \Delta^{cds} > \Delta^{nocds} \).

Under this Case 1.2, it is not possible to show analytically that \( \omega_{b}^{cds} \leq \omega_{b}^{nocds} \). Instead, I numerically verify this result and analytically characterize the conditions such that, \( \omega_{b}^{cds} \leq \omega_{b}^{nocds} \).

We saw from Case 1.1 that the sufficient condition (though not necessary) for \( \omega_{b}^{cds} \leq \omega_{b}^{nocds} \) is \( p_{b}^{cds} \geq p_{b}^{nocds} \). The equivalent of (A.26) without the CDS market is:

\[
(1 + \rho^{nocds}) \frac{F_h}{\gamma_d} = S \frac{\Delta \rho}{\gamma_d} + \frac{\Delta \rho}{\gamma_d} F_h - \frac{\Delta \rho}{\gamma_d} F_h + \frac{\Delta \rho}{\gamma_d} F_h - \frac{\Delta \rho}{\gamma_d} F_h
\]

(A.43)

Rearranging (A.26) in order to compare it to (A.43):

\[
(1 + \rho^{nocds}) \frac{F_h}{\gamma_d} = S \frac{\lambda_b \mu_{hn}}{\lambda_b \mu_{hn} + \gamma_d} + \mu_{hn} - \frac{\Delta \rho}{\gamma_d} \frac{F_h}{\gamma_d} - \frac{\lambda_c \mu_{hn}}{\lambda_c \mu_{hn} + \gamma_d + \gamma_u}
\]

(A.44)

The first two terms on RHS is increasing in \( \mu_{hn} \). Thus, the condition for \( \mu_{hn}^{cds} \geq \mu_{hn}^{nocds} \) is \( \Delta \rho \frac{F_h}{\gamma_d} \geq \frac{F_h}{\gamma_d} \frac{\lambda_b \mu_{hn}^{cds}}{\lambda_b \mu_{hn}^{cds} + \gamma_d + \gamma_u} \).

Case 2. Parameter conditions such that \( V_{hn}(\rho^{nocds} = 1) > O_h \).

From (A.26), keeping \( \rho \) fixed, \( \mu_{hn} \) is lower for some positive \( \lambda_c \) compared to the environment without CDS because high types end up selling CDS instead of buying bonds. As a result, \( \frac{x_b^{cds}(\gamma_d + r)}{b^{cds}(\gamma_d + r)} > \frac{x_b^{nocds}(\gamma_d + r)}{b^{nocds}(\gamma_d + r)} \). Combining it with the fact that the second term in (A.40) is positive, we get \( V_{hn}^{nocds} < V_{hn}^{cds} \) from (A.39) and (A.39). Thus, keeping entry fixed, the value of entering the economy as a high type is higher due to the increased trading opportunities. Since entry is already at the boundary \( \rho_{hn}^{nocds} = 1 \), entry cannot increase further, so the entry with the the CDS market is still \( \rho_{hn}^{cds} = 1 \). Thus, \( \mu_{hn}^{cds} < \mu_{hn}^{nocds} \).

From (A.35) and (A.36) and from previous arguments, the decrease in \( \mu_{hn} \) and the increase in \( V_{hn} \) both decrease \( V_{hab} \) and \( V_{aob} \). As a result, the marginal valuations of the bond \( (V_{hab} - V_{hn} \) and \( V_{aob}) \) are lower with the CDS market versus without. Thus, \( p_{b}^{nocds} > p_{b}^{cds} \) and \( \Delta^{cds} > \Delta^{nocds} \).

Case 3. Parameter conditions such that \( V_{hn}(\rho^{nocds} = 0) < O_h \).

We will consider three sub-cases.

Case 3.1. Parameter conditions such that \( V_{hn}(\rho^{nocds} = 0) < V_{hn}(\rho^{cds} = 0) < O_h \).

The solution with CDS is still a corner solution at \( \rho = 0 \). This is just the substitution effect and the results are the same as under Case 2.

Case 3.2. Parameter conditions such that \( V_{hn}(\rho^{nocds} = 0) < V_{hn}(\rho^{cds} = 0) = O_h \).

The solution with CDS is now an interior solution at \( \rho^{cds} \in (0, 1] \). Hence, \( \rho^{nocds} < \rho^{cds} \). The effect on the bond is analogous to Case 1.1: \( p^{cds} > p^{nocds} \), \( \Delta^{cds} > \Delta^{nocds} \).
Case 3.3 $V_{hn}(\rho^{nocds}) = 0 < O_h < V_{hn}(\rho^{cds})$.

The results will be analogous to Case 1.2. For $\lambda_c \geq \overline{\lambda}_c$, the complementarity effect dominates, while for $\lambda_c < \overline{\lambda}_c$ the substitution effect dominates as entry has reached its limit at $\rho^{cds} = 1$ and there is not enough entry.

Proof of Proposition [5] Consider what (A.26) limits to for an arbitrary $\lambda_b$ as $\lambda_c \to \infty$:

$$\frac{(1 + \rho)F_h}{\gamma_d} - \left(\frac{\lambda_b \mu_{hn}}{\gamma_d + \mu_{hn}} + \frac{F_l}{\gamma_u} + \mu_{hn}\right) = 0$$

(A.45)

The LHS of (A.45) is positive at $\mu_{hn} = 0$, decreasing in $\mu_{hn}$, and is negative for large $\mu_{hn}$. Thus, for any $\rho$, $\mu_{hn}$ is finite as $\lambda_c \to \infty$. As a result, $q_{aob}$, $q_{bs}$ and $q_{cb}$ are finite. Since $\mu_{hn} + \mu_{aoc} \to 0$, $q_{cb}$ is also finite. But $q_{cs} = \lambda_c \mu_{hn} \to \infty$. Thus, $\Delta_{hoc} \to 0$.

When the solution is interior,

$$V_{hn}^{cds} = V_{hn}^{nocds} = O_h$$

(A.46)

Then, using $\Delta_{hoc} \to 0$ and (A.30):

$$\frac{\lambda_b F_{bs} \phi_{hn}}{k^{cds}(\gamma_d + r)} = \frac{\lambda_b F_{bs} \phi_{hn}}{k^{nocds}(\gamma_d + r)}$$

(A.47)

Since this expression is uniquely determined by $\mu_{hn}$, it has to be that:

$$\mu_{hn}^{cds} = \mu_{hn}^{nocds}$$

(A.48)

Thus, $q_{cb} = \lambda_b \mu_{hn}$ is the same as without CDS. Consequently, from (A.35)–(A.36) and (A.46), $V_{hob}$ and $V_{aob}$ are the same with or without CDS. Thus, when $\lambda_c \to \infty$, the bond price is the same as in the benchmark environment without CDS. For (A.48) to hold, from (A.45), the entry rate (hence the measure of high types) increases enough to exactly offset the total measure of low types $\frac{F_l}{\gamma_u}$.

If entry is exogenous, $\lim_{\lambda_c \to \infty} p_h(\lambda_c) < p_h^{no$ cd$}$ because the measure of high types (hence the measure of bond buyers) decreases due the existence of low types.

Proof of Proposition [6] The population measures evolve according to:

\[\begin{align*}
\dot{\mu}_{hn}(t) &= (1 + \rho)F_h + \gamma_u \mu_{hoc}(t) - [\gamma_d \mu_{hn}(t) + (q_{bs}(t) + q_{cb}(t)) \mu_{hn}(t)] \\
\dot{\mu}_{ln}(t) &= F_l - [\gamma_u \mu_{ln}(t) + q_{cs} \mu_{hn}(t)] \\
\dot{\mu}_{hcb}(t) &= q_{bs} \mu_{hn}(t) - \gamma_d \mu_{hcb}(t) \\
\dot{\mu}_{aob}(t) &= \gamma_d \mu_{hcb}(t) - q_{bb} \mu_{aob}(t) \\
\dot{\mu}_{hoc}(t) &= q_{hc} \mu_{hn}(t) - [\gamma_d \mu_{hoc}(t) + \gamma_u \mu_{hoc}(t)] \\
\dot{\mu}_{aoc}(t) &= \gamma_d \mu_{hoc}(t) - [\gamma_u \mu_{aoc}(t) + q_{cs} \mu_{aoc}(t)] \\
\dot{\mu}_{lsc}(t) &= q_{cs} \mu_{hn}(t) - \gamma_u \mu_{lsc}(t)
\end{align*}\]
Value functions evolve according to:

\[
\begin{align*}
\dot{V}_{hn}(t) &= rV_{hn}(t) - [\gamma_d (0 - V_{hn}(t)) + q_{bs}(t) \omega_b(t) + q_{cb}(t)(V_{hoc}(t) - V_{hn}(t))] \quad (A.56) \\
\dot{V}_{ln}(t) &= rV_{ln}(t) - [\gamma_u (0 - V_{ln}(t)) + q_{cs}(t)(V_{ls}(t) - V_{ln}(t))] \quad (A.57) \\
\dot{V}_{hob}(t) &= rV_{hob}(t) - [\delta_b + x_b - y + \gamma_d (V_{aob}(t) - V_{hob}(t))] \quad (A.58) \\
\dot{V}_{aob}(t) &= rV_{aob}(t) - [\delta_b - y + q_{bh}(t)(1 - \phi) \omega_b(t)] \quad (A.59) \\
\dot{V}_{hoc}(t) &= rV_{hoc}(t) - [p_c(t) - (\delta_c - x_c) - y + \gamma_d (V_{aoc}(t) - V_{hoc}(t)) + \gamma_u (V_{hn}(t) - V_{hoc}(t))] \quad (A.60) \\
\dot{V}_{aoc}(t) &= rV_{aoc}(t) - [p_c(t) - \delta_c - y + q_{cs}(t)(0 - V_{aoc}(t)) + \gamma_u (0 - V_{aoc}(t))] \quad (A.61) \\
\dot{V}_{ls}(t) &= rV_{ls}(t) - [-p_c(t) + (\delta_c + x_c) - y + \gamma_u (0 - V_{ls}(t))] \quad (A.62)
\end{align*}
\]

Using the ODE for \(V_{hob}\) and \(V_{hn}\):

\[
\Delta_{hob} = r \Delta_{hob} - [\delta_b + x_b - y - (\gamma_d + q_{bs} \phi) \omega_b - q_{cb} \phi \omega_c]
\]

Together with the ODE for \(V_{aob}\):

\[
\dot{\omega}_b = -x_b + (r + \gamma_d + q_{bs} \phi + q_{bh}(1 - \phi)) \omega_b + q_{cb} \phi \omega_c \quad (A.63)
\]

Analogously, we get the ODE for \(\omega_c\),

\[
\dot{\omega}_c = -x_c + q_{bs} \phi \omega_b + (r + \gamma_d + \gamma_u + q_{cb} \phi + q_{cs}(1 - \phi)) \omega_c \quad (A.64)
\]

To solve for \(\omega_b\) and \(\omega_c\), we write (A.63) and (A.64) in this form:

\[
\begin{bmatrix}
\dot{\omega}_b(t) \\
\dot{\omega}_c(t)
\end{bmatrix} = - \begin{bmatrix}
x_b \\
x_c + x_c - 2y
\end{bmatrix} + A(t) \begin{bmatrix}
\omega_b(t) \\
\omega_c(t)
\end{bmatrix},
\]

where

\[
A(t) = \begin{bmatrix}
r + \gamma_d + q_{bs} \phi + q_{bh}(1 - \phi) & q_{cb} \phi \\
q_{bs} \phi & r + \gamma_d + \gamma_u + q_{cb} \phi + q_{cs}(1 - \phi)
\end{bmatrix}
\]

Thus, the solution is:

\[
\begin{bmatrix}
\omega_b(t) \\
\omega_c(t)
\end{bmatrix} = \int_t^\infty e^{-\int_s^t A(u) \, du} \begin{bmatrix}
x_b \\
x_c + x_c - 2y
\end{bmatrix} \, ds
\]

From here, the solutions to the ODE for \(\Delta_{hob}\) and \(V_{aob}\) are given by:

\[
\Delta_{hob} = \frac{\delta_b + x_b - y}{r} - \int_t^\infty e^{-r(s-t)} ((\gamma_d + q_{bs} \phi) \omega_b(t + q_{cb} \phi \omega_c) \, ds
\]

\[
V_{aob} = \frac{\delta_b - y}{r} + \int_t^\infty e^{-r(s-t)} q_{bb} (1 - \phi) \omega_b(t) \, ds
\]

\[\square\]

### A.1 A Simple Example of Hedging Benefits

Let \(\theta = 1\) denote a long position (exposed to risk) through the bond or CDS market, \(\theta = 0\) no position, and \(\theta = -1\) a short position (i.e. bought CDS). An agent with \(\theta_b \in \{0, 1\}\) shares of the
bond has a utility flow

$$\theta_b \left( \delta_b + x_t^b \right)$$

and an agent with CDS position $\theta_c \in \{-1, 0, 1\}$ has a utility flow:

$$- \theta_c (\delta_c + x_t^c)$$

where $x_t^b \in \{-x_b, 0, x_b\}$ and $x_t^c \in \{-x_{ch}, 0, x_{cd}\}$ are stochastic processes. I define an agent with $\{x_t^b = x_b, x_t^c = -x_{ch}\}$ as a high type, with $\{x_t^b = 0, x_t^c = 0\}$ as an average, and with $\{x_t^b = -x_b, x_t^c = x_{cd}\}$ as a low type.

The bond coupon flow, $\delta_b$, can be interpreted as an expected coupon flow: with intensity $\eta$ the bond defaults but otherwise pays $1$ of coupon. Hence, $\delta_b = (1 - \eta)\$1$. Similarly, $\delta_c$ can be interpreted as an expected insurance payment. A CDS contract pays out if there is default on the coupon payment: with intensity $\eta$ CDS pays $\$1$ thus, $\delta_c = \eta\$1$. According to (A.65), a high type values this as $\delta_c - x_{ch}$, while a low type values this as $\delta_c + x_{ch}$. Thus, as a CDS seller ($\theta_c = -1$), a low type experiences a greater disutility paying out the insurance payment $-(\delta_c + x_{cd})$ than a high type $-(\delta_c - x_{ch})$. Conversely, as a CDS buyer ($\theta_c = 1$), a low type benefits more receiving the insurance payment $(\delta_c + x_{cd})$ than a high type $(\delta_c - x_{ch})$.

Table 4: The Expected Valuation of the Bond Payoff

Consider an example where with intensity, $\eta$, the bond defaults and pays no coupon, otherwise pays $\$1$ of coupon. Hence, the expected coupon is $\delta_b = (1 - \eta)\$1$. The table shows valuations of the bond cash flow by high, average, and low types.

<table>
<thead>
<tr>
<th>Bond Payoff</th>
<th>Utility Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - \eta$</td>
<td>$$1$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$$0$</td>
</tr>
<tr>
<td>$\delta_b$</td>
<td>$\epsilon_h$</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>$-\epsilon_l$</td>
</tr>
<tr>
<td>Expected Valuation:</td>
<td>$\delta_b$ $(1 - \eta)$ $\delta_c$ $(1 - \eta)\frac{x_{ch}}{\eta} \delta_c$ $(1 - \eta)\frac{x_{cd}}{\eta}$</td>
</tr>
</tbody>
</table>

Table 5: The Expected Valuation of the CDS Payoff as a CDS Buyer

With intensity $\eta$ the bond defaults and CDS pays $\$1$ and zero otherwise. Hence, the expected insurance payment is $\delta_c = \eta\$1$. The table shows a simple example of utility valuations of the cash flow as a CDS buyer by different types. In the default state, low types get an extra utility for extra $\$1$ than high types. Thus, in expectation, as a CDS buyer a low type benefits more receiving the insurance payment $(\delta_c + x_{cd})$ than a high type $(\delta_c - x_{ch})$.

<table>
<thead>
<tr>
<th>Cash Flow of CDS Buyer</th>
<th>Utility Valuations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - \eta$</td>
<td>$$0$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$$1$</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>$1 - \epsilon_h$</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>$1 + \epsilon_l$</td>
</tr>
<tr>
<td>Expected Valuation:</td>
<td>$\delta_c$ $\frac{x_{ch}}{\eta}$ $\delta_c$ $\frac{x_{cd}}{\eta}$</td>
</tr>
</tbody>
</table>

---

24 For an expository purpose, let us ignore $y$ that is in section 1.

25 The Online Appendix is available at https://sites.google.com/site/sambalaibat/
Table 6: The Expected Valuation of the CDS Payoff as a CDS Seller

With intensity \( \eta \) the bond defaults and CDS seller has to pay $1. The table shows a simple example of utility valuations of the cash flow as a CDS seller by different types. In the default state, low types get an extra disutility for paying out the insurance than high types. Thus, in expectation, as a CDS seller a low type experiences a greater disutility paying out the insurance payment \( -(\delta_c + x_{cl}) \) than a high type \( -(\delta_c - x_{ch}) \).

<table>
<thead>
<tr>
<th>Cash Flow of CDS Seller</th>
<th>Utility Valuations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>( 1 - \eta )</td>
<td>$0</td>
</tr>
<tr>
<td>( \eta )</td>
<td>-$1</td>
</tr>
<tr>
<td>Expected Valuation</td>
<td>( -(\frac{\delta_c}{\eta} - \frac{x_{ch}}{\eta \epsilon_h}) )</td>
</tr>
</tbody>
</table>

B Appendix: Model Figures

Figure 2: The Bond Illiquidity Discount

This figure illustrates the main result of the paper. It shows the difference in the bond illiquidity discount with and without CDS as a function of the CDS market efficiency \( \lambda_c \). With the existence of naked CDS buyers, the bond illiquidity discount (the solid blue line) is lower than the benchmark without CDS (dashed red line). If the CDS market is frictionless \( \lambda_c \rightarrow \infty \), the CDS market is redundant and does not affect bond market liquidity.
Figure 3: The Value of Trading as a High Type

The figure plots the value of trading as a long trader, $V_{hn}$, as a function of the entry rate ($\rho$). The value increases with the introduction of the CDS market (the curve shifts up) and is higher with search frictions present in the CDS market ($\lambda_c < \infty$) than without search frictions in the CDS market ($\lambda_c = \infty$). The equilibrium entry rate is determined by the intersection of $V_{hn}$ and their outside option (the horizontal line at $O_h$).

Figure 4: The Rate of Entry

The diagram illustrates how the introduction of the CDS market affects the entry rate, $\rho$, of long traders. By how much the entry rate increases depends on the total potential demand for CDS (i.e. the steady state measure of low types, $F_{\gamma u}$, who in equilibrium want to short credit risk) and the CDS market matching efficiency, $\lambda_c$. The dashed line is the additional number of long traders in the economy due to the existence of naked CDS buyers. As the CDS market is frictionless, $\lambda_c \to \infty$, the increase in the measure of long traders exactly equals the demand for CDS (the horizontal line).
Figure 5: The Effect of CDS on Bond Market Composition and Bond Volume

The two left panels compare the relative composition of buyers and sellers in the bond market with (solid line) and without CDS (dashed line) as a function of the CDS market efficiency ($\lambda_c$). The introduction of the CDS market increases the number of bond buyers (the left panel) and decreases the number of bond sellers (the middle panel). If the CDS market is frictionless ($\lambda_c \to \infty$), the CDS market is redundant and does not affect the bond market composition. The right panel compares the volume of trade in the bond market with (solid line) and without CDS (dashed line) as a function of the CDS market efficiency ($\lambda_c$). The introduction of the CDS market increases bond market volume. If the CDS market is frictionless ($\lambda_c \to \infty$), the CDS market is redundant and does not affect bond market volume.

Figure 6: The Transition Dynamics of Types’ Measures After a Temporary CDS Ban

A temporary naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets the number of naked CDS buyers to zero (as can be seen in the left panel). The figure plots the time varying equilibrium path back to the steady state number of CDS buyers (the left panel), bond buyers (the middle panel), and bond sellers (the right panel).

Figure 7: The Transition Dynamics of Bond Illiquidity

A temporary naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets the number of naked CDS buyers to zero. The figure plots the short run dynamics of the bond illiquidity discount. With a temporary naked CDS ban, the illiquidity discount temporarily decreases (i.e. liquidity increases).

The Bond Illiquidity Discount
Figure 8: Cost of Entry

\[ V_{hn}(\rho) \]

\[ \rho_{perm} \quad \rho_{tem} \quad \rho_{cds} \]

with cds

temporary ban, \( t = 0 \)

permanent ban

\[ O_h + c(\rho) \]

Figure 9: The Implicit Short-run Dynamics of the Cost of Entry \( c(\rho(t)) \)

\[ t = 0 \quad t \]
Figure 10: The Permanent EU Naked CDS Ban and the Amount of CDS Purchased, 2011.01 - 2012.08
The solid line plots the total CDS purchased (CDS net notional, $bln) across countries that were subject to the EU ban. The dashed line plots the total for countries that were not affected by the ban and CDS could still be purchased. The vertical line is drawn at October 18, 2011 and shows when the EU passed the naked CDS ban legislation.

Figure 11: The Permanent EU Naked CDS Ban and Bond Illiquidity, 2011.01 - 2012.08
The vertical line drawn at October 18, 2011 shows when the EU passed the naked CDS ban legislation. The solid line plots the cross-country average bond bid-ask spread (% of the mid price) for the countries subject to the ban (the EU countries). The dashed line plots the average bond bid-ask spread for countries that were not affected by the ban (outside the EU). We see that the countries affected by the ban experienced an increase in their bond bid-ask spread.
Figure 12: The Temporary CDS Ban and Bond Illiquidity, Mar 2010 - Aug 2010
The solid line plots the cross-country average bond bid-ask spread (% of the mid price) for the EU countries that were subject to the ban (i.e. Eurozone countries). The dashed line shows the average for the EU countries not affected by the ban (i.e. naked CDS referencing these countries could still be purchased). The vertical lines are drawn at the week before and after the German ban is instituted. We see that the countries affected by the ban experienced an immediate decrease in their bond bid-ask spread.

Figure 13: The Temporary German CDS Ban and the Amount of CDS Purchased, Mar 2010 - Aug 2010
The solid line plots the time series of the total CDS net notional (billion) across EU countries that were subject to the ban (i.e. Eurozone countries). The dashed line shows the total for EU countries that were not affected by the ban (i.e. naked CDS referencing these countries could still be purchased). The vertical lines are drawn at the week before and after the German ban is instituted.
<table>
<thead>
<tr>
<th>The German Banking Industry Committee:</th>
</tr>
</thead>
<tbody>
<tr>
<td>“The market has become less liquid; the bid-offer spread has widened. Volatility is unchanged, but has tended to shift to the spot/cash markets.”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Association for Financial Markets in Europe (AFME) and the International Swaps and Derivatives Association (ISDA):</th>
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</thead>
<tbody>
<tr>
<td>“Market participants have already observed that seemingly due to the SSR Regulation (restrictions it imposed on sovereign debt and sovereign CDS markets), Asian participation in the European bond market fell by around 50% immediately after the introduction of the SSR, thus demonstrating neatly one adverse impact of the SSR in general in driving investors away.”</td>
</tr>
<tr>
<td>“Some buy side market participants have already remarked that even though there is still liquidity in sovereign debt, it is more difficult to source this liquidity.”</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternative Investment Management Association (AIMA) and Managed Funds Association (MFA):</th>
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</thead>
<tbody>
<tr>
<td>“Some of our members have reported that they have stopped trading European sovereign CDS and bonds, given the regulatory and reputational risks.”</td>
</tr>
<tr>
<td>“Restrictions on CDS positions over the medium term will generally make it more difficult for sovereign issuers to borrow through long-dated securities, leading to a shortening of the average maturity profile of sovereign issuance as investors seek to limit their risk exposure, thereby increasing the vulnerability of sovereigns to short term liquidity and funding crises. This sentiment is reflected in the responses to AIMA and MFA’s poll of their members.”</td>
</tr>
<tr>
<td>“At worst, the ban could ultimately undermine liquidity in the underlying sovereign debt markets, undermining the ability of sovereigns to raise finance through debt issuance.”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deutsche Bank</th>
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<tbody>
<tr>
<td>“We observed anecdotally that as investors began to understand the details of the regulation, cash volumes reduced with a resultant increase in volatility, although this was not significant.”</td>
</tr>
</tbody>
</table>
References


Delatte, Anne Laure, Mathieu Gex, and Antonia López-Villavicencio, 2011, Has the CDS market influenced the borrowing cost of European countries during the sovereign crisis?, *Working Paper*.


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